Math 4. Class Work 13

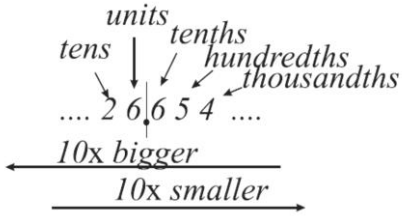
Decimals

To represent numbers smaller than one – use fractions or decimal numbers.

- The clock system (60 base system) – measuring time and angles
- The 10-base place value system – divide 1 (a unit) into 10 parts, then into 10 parts again …. Can write smaller and smaller numbers.

We get series of fractions with denominators 1/10 , 1/100, 1/1000 … This is $26\ and\ \frac{6}{10}$

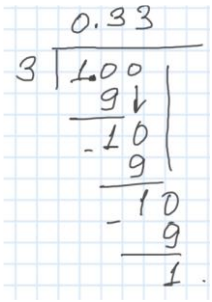
Example: $\mathbf{26.654} = 10 \cdot 2 + 1 \cdot 6 + \frac{1}{10} \cdot 6 + \frac{1}{100} \cdot 5 + \frac{1}{1000} \cdot 4 = 10 \cdot 2 + 1 \cdot 6 + \frac{6}{10} \cdot + \frac{5}{100} + \frac{4}{1000} =$
$10 \cdot 2 + 1 \cdot 6 + \frac{600}{1000} + \frac{50}{1000} + \frac{4}{1000}$

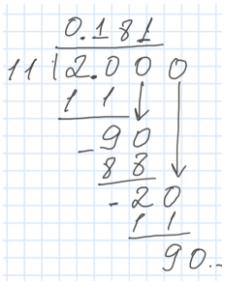


This number can be expressed as a mixed fraction with a denominator 1000, …, but in decimal notation, all arithmetic operations are much easier to perform.

Converting a fraction into a decimal

1. Use long division – divide the numerator by the denominator





$\frac{1}{3} = 1:3 = 0.3333 \ldots = 0.\overline{3}$ $\frac{2}{11} = 2:11 = 0.1818 \ldots = 0.\overline{18}$ $\frac{3}{5} = 3:5 = 0.6$

an infinite decimal, an infinite periodical decimal, a finite decimal

2. Convert the denominator to 10, 100, 1000 … If the fraction's denominator can be prime factorized into products of 2 and/or 5 only!

Examples: $0.3 = \frac{3}{10}$; $0.27 = \frac{2}{10} + \frac{7}{100} = \frac{27}{100}$; $0.75 = \frac{75}{100} = \frac{3 \cdot 25}{4 \cdot 25} = \frac{3}{4}$

$$\frac{1}{25} = \frac{1}{5 \cdot 5} = \frac{1}{5 \cdot 5} = \frac{1 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 2 \cdot 2} = \frac{4}{10 \cdot 10} = \frac{4}{100} = 0.04$$

$$\frac{7}{8} = \frac{7}{2 \cdot 2 \cdot 2} = \frac{7 \cdot 5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} = \frac{875}{1000} = 0.875$$

Multiplication/division of decimals

- To multiply any natural number by 10, we need to write 0 at the end of a number, increasing all place values 10 times. We can prove this using the distributive property

$$\mathbf{245} \cdot 10 = (100 \cdot 2 + 10 \cdot 4 + 5) \cdot 10 = 100 \cdot 10 \cdot 2 + 10 \cdot 10 \cdot 4 + 10 \cdot 5$$
$$= 1000 \cdot 2 + 100 \cdot 4 + 10 \cdot 5 + 1 \cdot 0 = 245\mathbf{0}$$

- The same applies when multiplying decimals; multiplied by 10, or 100, or 1000 means – move the decimal point that many positions to the right as the zeros (increase all place values 10 times). Multiplying by 100 – move the point 2 positions to the right…

$$\mathbf{245.23} \cdot 10 = (100 \cdot 2 + 10 \cdot 4 + 5 + 0.1 \cdot 2 + 0.01 \cdot 3) \cdot 10$$
$$= 100 \cdot 10 \cdot 2 + 10 \cdot 10 \cdot 4 + 10 \cdot 5 + 0.1 \cdot 10 \cdot 2 + 0.01 \cdot 10 \cdot 3 =$$
$$= 1000 \cdot 2 + 100 \cdot 4 + 10 \cdot 5 + 1 \cdot 2 + 0.1 \cdot 3 = \mathbf{2452.3}$$

What about dividing by 10? Move the point to the left (decrease all place values by 10)

$$235 : 10 = 235 \cdot \frac{1}{10} = (100 \cdot 2 + 10 \cdot 3 + 1 \cdot 5) \cdot \frac{1}{10} = \frac{100}{10} \cdot 2 + \frac{10}{10} \cdot 3 + \frac{1}{10} \cdot 5 = 20 + 3 + \frac{5}{10}$$
$$= 23.5$$

To perform the long multiplication of the decimals, we do the multiplication procedure as we would do with natural numbers, regardless of the position of decimal points. At the end, the decimal point should be placed on the resulting line as many steps from the right side as the *sum of the decimal digits of both numbers*.

$$38.6 \cdot 5.78 = 38.6 \cdot \mathbf{10} \cdot 5.78 \cdot \mathbf{100} : (10 \cdot 100) = 386 \cdot 578 : 1000$$

1. Read the numbers
   a) 7. 4
   b) 11. 03
   c) 324. 73
   d) 0.0686

2. Write the numbers as decimals.
   a) Twenty-four and six tenths
   b) Three hundred and two and twenty-four thousandths
   c) Eight hundredths

3. Write in decimal notation the following fractions:
   Example:

$$1\frac{3}{25} = 1 + \frac{3}{25} = 1 + \frac{3 \cdot 4}{25 \cdot 4} = 1 + \frac{12}{100} = 1 + 0.12 = 1.12$$

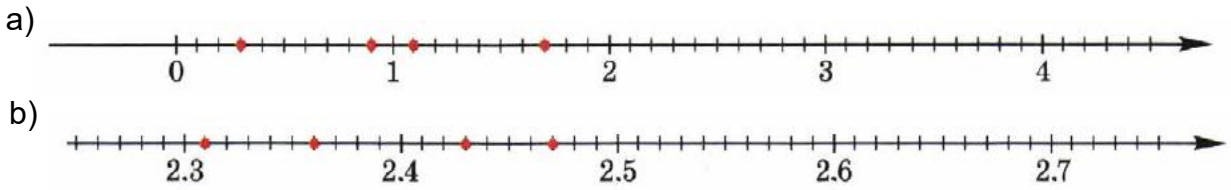
$1\frac{1}{10}$ $\quad$ $2\frac{4}{10}$

$24\frac{25}{100}$ $\quad$ $98\frac{3}{100}$

$4\frac{333}{1000}$ $\quad$ $8\frac{45}{1000}$

$75\frac{8}{10000}$ $\quad$ $9\frac{565}{10000}$

4. Which numbers are marked on the number lines below:



5. Evaluate in the most convenient way:

a. $1.2 + 2.3 + 3.4 + 4.5 + 5.6 =$

b. $2.3 + 3.4 + 4.5 - 5.6 + 9.2 =$

6. On graph paper, draw a number line; use 10 squares as a unit. Mark points with coordinates 0.1, 0.5, 0.7, 1.2, 1.3, 1.9.

7. Which fractions below can be written in as a finite decimal:

   $\mathbf{\frac{1}{2}}$, $\mathbf{\frac{4}{3}}$, $\frac{1}{4}$, $\mathbf{\frac{12}{5}}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$,

   $\frac{1}{11}$, $\frac{1}{12}$, $\frac{1}{13}$, $\frac{1}{14}$, $\frac{1}{15}$, $\frac{1}{16}$, $\frac{1}{17}$

   Why do you think so?

8. Write decimals as fractions and evaluate the following expressions:

   a. $\frac{2}{3} + 0.5$ *b.* $\frac{1}{3} \cdot 0.9$ *c.* $\frac{3}{16} \cdot 0.16$

   d. $0.6 - \frac{2}{5}$ *e.* $0.4 : \frac{2}{7}$; *f.* $\frac{9}{20} : 0.03$

9. Which part of 1 m is 1 cm?
   Which part of 1 km is 1 m?
   Which part of 1 cm is 1 mm?
   Which part of 1 m is 1 dm?
   Which part of 1 kg is 1 g?
   Which part of 1 g is 1 mg?

10. 1 kilogram of candies costs 16 dollars. How much

    a. 2 kg will cost?
    b. 0.5 kg will cost?
    c. 1.2 kg will cost?
    d. 0.75 kg will cost?
    e. 0.4 kg will cost?
    f. 2.5 kg will cost?