Math 5e, Fall 2024 Homework 13

Homework #13 is due January 8

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. Write on separate sheets of paper, not between the lines of this handout!

General notation (n is a whole number):

$$a^n = a \times a \times a \times \ldots \times a \ (n \text{ times})$$

Special cases:

$a^0 = 1$	read: a-to-the-zero
$a^1 = a$	is just itself 'a'
$a^2 = a \times a$	read: a-squared
$a^3 = a \times a \times a$	read: a-cubed

Product $a^n a^m = a^{n+m}$

Division $\frac{a^n}{a^m} = a^{n-m}$

$a^n = \frac{1}{a^{-n}}$ and $a^{-n} = \frac{1}{a^n}$

Power of a product $(ab)^n = a^n \times b^n$

Power raised to a power $(a^m)^n = a^{m \times n}$

Scientific notation

- Scientific notation is a convenient way to write very large numbers: instead of writing 2 000 000 000 one can say "2 and then 9 zeros". Since writing a zero at the end is the same as multiplication by 10, we can also write the same number as $2 \times 10 \times \ldots . \times 10$ (9 $times$) or, for short, $2 \times 10^{9,}$ which is much shorter.

Such a form (a decimal with one digit before the decimal point, times 10 to some power) is called *scientific notation*.

- To write a number larger than 10 in scientific notation, you should:
  1. Count how many digits the whole part has. The power of 10 will be that number of digits minus 1.
  2. Write down the digits of the number, but now put the decimal point after the first digit.

Example: $3{,}412{,}000 = 3.412000 \times 10^6 = 3.412 \times 10^6$

- In a similar way, scientific notation is very useful for very small numbers.

For example, the weight of one atom of hydrogen is about 1.66×10^{-24}grams — or

0.000000000000000000000000166 gr

For the homework, let me remind you that you should write full solutions and show all your calculations and reasoning – not just answers! Do not write on this homework assignment – use a separate sheet of paper instead; leave the homework assignment in your folder for future reference.

Homework problems on the next page

1. If $a = 2^{-13}3^{9}$ and $b = 2^{11}3^{-7}$what is the value of *ab*? of *a/b*?

2. How many zeroes does the number $4^{15}5^{26}$ end with?

3. About how many hydrogen atoms are there in one gram of hydrogen? See the first page for the weight of one hydrogen atom.

4. Write the following numbers using scientific notation.
   a. the distance from Earth to Pluto is ≈ 7,527,000,000 km;
   b. the distance from Earth to the star Sirius is ≈ 81,900,000,000,000 km;
   c. the distance from Earth to Vega is ≈ 249,500,000,000,000 km;
   d. the distance from Earth to the Andromeda Nebula is ≈ 2,000,000,000,000,000,000,000 km;
   e. the area of the Pacific Ocean is ≈ 178,684,000,000 km^2

5. Write the following numbers in regular form (no powers):

   (a) 9.21×10^6 = (b) 1.527×10^4 =

   (c) 5.3459×10^3 = (d) 7.527×10^2 =

6. Perform calculations in the base-4:

$$\begin{array}{r} 111 \\ +222 \\ \hline \end{array} \qquad \begin{array}{r} 321 \\ +123 \\ \hline \end{array} \qquad \begin{array}{r} 310 \\ +\ 23 \\ \hline \end{array}$$

7. (*No written solution required, not graded*) Towers of Hanoi.
   Legend has it that a group of Eastern monks are the keepers of three towers on which sit 64 golden rings. Originally, all 64 rings were stacked on one tower, with each ring smaller than the one beneath. The monks are to move the rings from this first tower to the third tower one ring at a time but never moving a larger ring on top of a smaller one. Once the 64 rings have all been moved, the world will come to an end.

   Can you suggest a strategy to do this, first using smaller numbers (e.g., start with just 2 rings, then 3 rings, then 4)? You can use disks from a baby toy pyramid, or you can play this game online at:
   http://www.mathsisfun.com/games/towerofhanoi.html