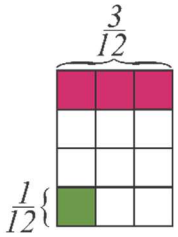
Math 4. Class Work 6

Fractions

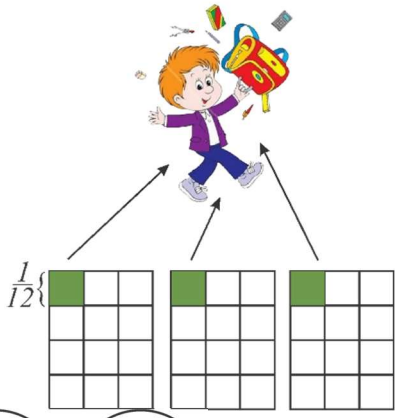
A fraction (from Latin – broken) represents a part of a whole.

A whole chocolate bar is divided into 12 equal pieces:



$$1\ (whole\ chocolate\ bar): 12(equal\ parts) = \frac{1\ (whole\ chocolate\ bar)}{12(equal\ parts)}$$

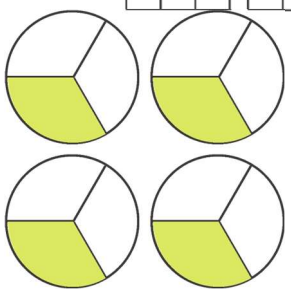
$$= \frac{1}{12}(of\ the\ whole\ chocolate\ bar, green\ part)$$



To divide 3 chocolate bars among 12 kids, we can give each kid $\frac{1}{12}$ of each chocolate bar, then one kid will get altogether

$$\frac{1}{12}+\frac{1}{12}+\frac{1}{12}=3\times\frac{1}{12}=\frac{3}{12}=\frac{1}{4}\ (in\ red)$$

$$\text{Or } 3\div 12=3\times\frac{1}{12}=\frac{3}{12}=\frac{1}{4}$$

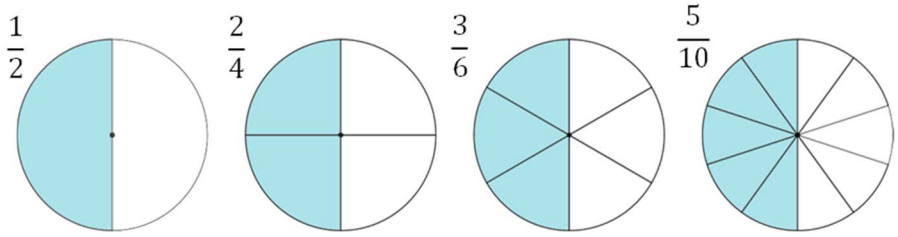


Mixed fractions (whole number + fraction)

To divide 4 pizzas equally between 3 friends, we will give each friend $\frac{1}{3}$ of each pizza. Each friend will get $4\div 3=4\times\frac{1}{3}=\frac{4}{3}$, which is exactly 1 whole pizza $(3\times\frac{1}{3}=\frac{3}{3}=1)$ and $\frac{1}{3}$.

Equivalent fractions.

Some fractions can look different but represent exactly the same part of the whole.



$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{5}{10};$$

We are only dividing the whole into smaller parts and taking more such parts

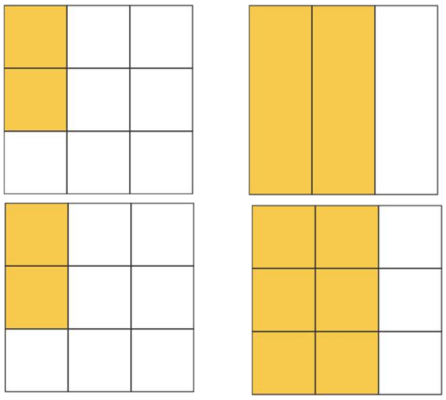
$$\frac{1}{2}=\frac{1\cdot 2}{2\cdot 2}=\frac{1\cdot 3}{2\cdot 3}=\frac{1\cdot 5}{2\cdot 5}$$

We can multiply the numerator and denominator of a fraction by the same number: if the parts are twice smaller (the denominator is multiplied by 2), we need twice as many such parts to keep the fraction the same (the numerator is

Simplifying fractions: This property of fractions can be used to reduce fractions. If there are common factors in the numerator and denominator, both numbers can be divided by common factors.

$$\frac{25}{35} = \frac{5 \cdot 5}{7 \cdot 5} = \frac{5}{7}; \qquad \frac{77}{352} = \frac{7 \cdot 11}{32 \cdot 11} = \frac{7}{32}$$

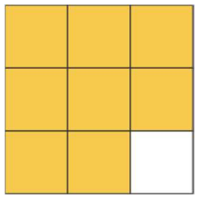
Addition and subtraction of fractions with unlike denominators.



We can only add similar objects: apples and oranges to oranges. To add two fractions, we must be sure they have **the same denominator.**

For example: $\frac{2}{9} + \frac{2}{3} = \frac{2}{9} + \frac{6}{9} = \frac{8}{9}$

because $\frac{2}{3}$ is precisely the same as $\frac{2}{3} = \frac{2 \cdot \mathbf{3}}{3 \cdot \mathbf{3}} = \frac{6}{9}$



Common denominator

- The common denominator of fractions should be the multiple of these denominators – the **LCM** can do this task!

For example, $\frac{3}{8} + \frac{5}{12}$

denominator can be $8 \cdot 12 = 96$, but 24 is smaller $\frac{3 \cdot 3}{8 \cdot 3} + \frac{5 \cdot 2}{12 \cdot 2} = \frac{9}{24} + \frac{10}{24} = \frac{19}{24}$

- If both numbers are prime (or mutually prime), the least common multiple is their product.

Compare fractions: find if one fraction is greater, smaller, or equal to another.

- First, fractions can be brought to a common denominator and compare the numerators

For example, let's compare $\frac{7}{12}$ and $\frac{10}{18}$. The first fraction, $\frac{7}{9}$, is non-reducible. The second fraction can be reduced (reduce fractions before doing anything else): $\frac{10}{18} = \frac{2 \cdot 5}{2 \cdot 9} = \frac{5}{9}$

Bring to a common denominator: $\frac{7}{12} = \frac{7 \cdot 3}{12 \cdot 3} = \frac{21}{36}; \quad \frac{5}{9} = \frac{5 \cdot 4}{9 \cdot 4} = \frac{20}{36}; \quad \frac{\mathbf{21}}{36} > \frac{\mathbf{20}}{36}$

The whole was divided into 36 equal parts, and 21 such parts are greater than 20.

- Another way is to bring fractions to a common numerator and compare the denominators.

Since 5 and 7 are both prime numbers, the LCM of them is their product:

$$\frac{7}{12}=\frac{7\cdot 5}{12\cdot 5}=\frac{35}{60};\quad \frac{5}{9}=\frac{5\cdot 7}{9\cdot 7}=\frac{35}{63};\quad \frac{35}{\mathbf{60}}>\frac{35}{\mathbf{63}}$$

An equal number of parts are compared, 35, but each part in the second case is smaller.

- Also, both fractions can be compared with a third number; for example, $\frac{1}{2}$.

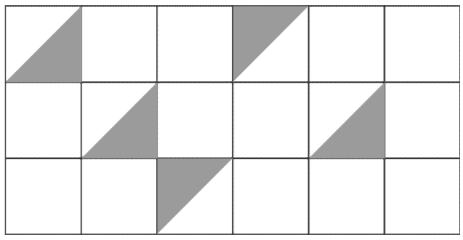
$$\frac{7}{12}=\frac{6}{12}+\frac{1}{12}=\frac{1}{2}+\frac{\mathbf{1}}{\mathbf{12}};\quad \frac{10}{18}=\frac{9}{18}+\frac{1}{18}=\frac{1}{2}+\frac{\mathbf{1}}{\mathbf{18}}$$

Since, $\frac{7}{12}$ is greater than $\frac{1}{2}$ by $\frac{1}{12}$; and $\frac{10}{18}$ is greater than $\frac{1}{2}$ by $\frac{1}{18}$, $\frac{1}{12}>\frac{1}{18}$, so $\frac{7}{12}>\frac{10}{18}$.

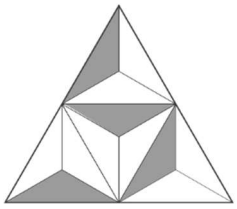
Problems:

1. Write a fraction which show the shaded part of the shapes:

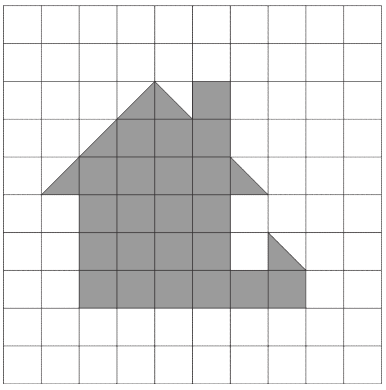
a.



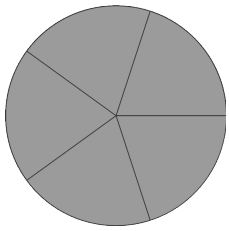
b.



c.

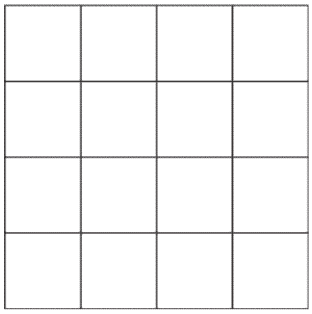


d.

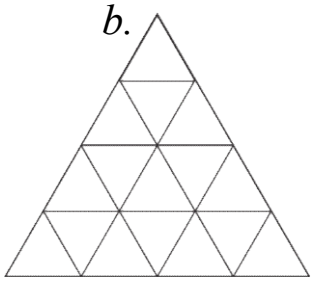


2. Shade the corresponding part of the figure: $\frac{3}{8}$, $\frac{4}{16}$, $\frac{3}{15}$

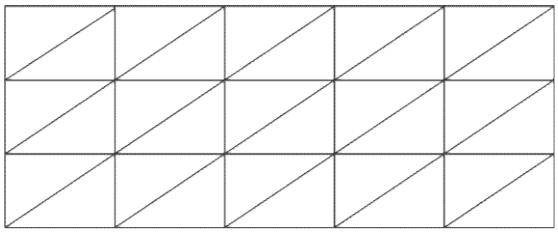
a.



b.

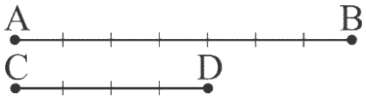


c.

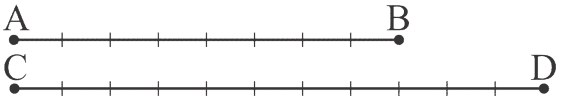


3. What part of the segment [AB] is the segment [CD]?

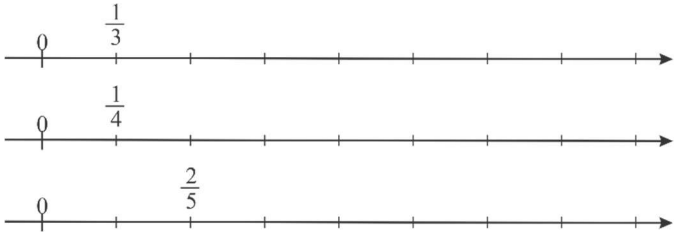
a.



b.



4. On the number lines, mark the number 1.



5. Draw a number line with a unit segment equal to 10 cells and mark the fractions:

$$\frac{1}{5}; \frac{2}{5}; \frac{3}{5}; \frac{4}{5}; \frac{5}{5}; \frac{6}{5}; \frac{7}{5}; \frac{8}{5}.$$

6. Fill in the empty spaces in the following fractions:

$$\frac{2}{3} = \frac{\square}{9} = \frac{\square}{21} = \frac{4}{\square} = \frac{36}{\square}$$

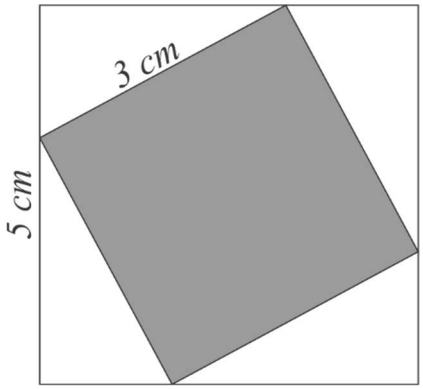
7. Evaluate:

a. $\frac{1}{5} + \frac{1}{2}$; *b.* $\frac{2}{5} + \frac{3}{10}$; *c.* $\frac{5}{9} - \frac{1}{3}$;

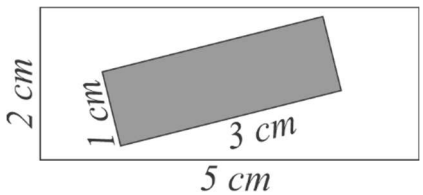
8. Compare (use the rules to compare fractions):

a. $\frac{3}{5}$ *and* $\frac{2}{5}$; *b.* $\frac{3}{5}$ *and* $\frac{3}{8}$ *c.* $\frac{3}{6}$ *and* $\frac{1}{2}$

9. What part of the
   a. big square the shaded square is?



   b. big rectangle the shaded rectangle is?



10. Write the answer as a fraction.

a. Milk was evenly poured into 6 glasses. What fraction of the milk is in 1 glass? In 3 glasses? In 5 glasses?

b. In a bundle of 11 balloons: 3 of them are yellow, 4 are green, the rest are red. What fraction of all the balloons are red? Yellow? Green?

11. Find the part of the whole

$a.\ \frac{3}{4}\ of\ 12,$

$b.\ \frac{2}{7}\ of\ 14,$

$c.\ \frac{5}{8}\ of\ 56$

12. In the school cafeteria, there are 12 tables. There are 10 seats at each table. At lunchtime $\frac{4}{5}$ of all seats were occupied by students. How many students were in the cafeteria?

13. An apple worm was eating an apple. On the first day, it ate half of the apple, on the second day it ate half of the rest, and on the third day it ate half of the rest again. On the fourth day it ate all the leftovers. What part of the apple did it eat on the fourth day?

14. Peter spent 2 hours doing his homework. $\frac{1}{3}$ of this time, he spent doing his math homework and $\frac{1}{4}$ of the remaining time he spent on the history assignment. How many minutes did Peter spend on his history assignment, and how many minutes did he spend doing his math homework?

15. Half of the students in the class participated in a spelling bee competition. One-third of them became winners. How many students are in the class if there are 5 winners of the spelling bee in the class?