Math 4. Class Work 5

Prime factorization.

> Natural numbers greater than 1 that have no divisors other than 1 and themselves, are called **prime numbers**.

Prime numbers: 2, 3, 5, 7, 11, 13, …..   2 is the only even prime number

Prime factors of $168 = 2, 2, 2, 3, 7$

Prime factorization process:

$$
\begin{array}{l}
168 \div 2 \\
84 \div 2 \\
42 \div 2 \\
21 \div 3 \\
7 \div 7 \\
1
\end{array}
$$

The Greatest Common Factor or Divisor (GCF) of two numbers, is the largest number that can be a divisor for both numbers.

Example: $GCF(168, 180) = 12$.
Find the prime factorization for both numbers, find the multiple common in both numbers

$$2 \cdot 2 \cdot 2 \cdot 3 \quad \cdot 7 = 168 \qquad\qquad \Rightarrow\ 2 \cdot 2 \cdot 3 = 12$$

$$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \quad = 180$$

The Least Common Multiple (LCM) of two numbers is the smallest number that is divisible by both of these numbers.

Example: $LCM(12, 15) = ?$ (smallest number divisible by both 12 and 15)
Find the prime factorization of 12 and 15:

$$
\begin{array}{c|c}
12 & 2 \\
6 & 2 \\
3 & 3 \\
1 &
\end{array}
\qquad
\begin{array}{c|c}
15 & 3 \\
5 & 5 \\
1 &
\end{array}
\quad \Rightarrow \quad
\begin{array}{l}
2 \cdot 2 \cdot 3 = 12 \\[4pt]
3 \cdot 5 = 15
\end{array}
\quad \Rightarrow \quad
2 \cdot 2 \cdot 3 \cdot 5 = 60
$$

The number we are looking for has to be a product of prime factors 12 and 15. As 3 is common, the LCM is a product of all factors in both without repeat, this number is 60.

Problems:

Warm -up: Find all divisors of

   6;   7;   14;   18;   70;   60.

1. Without calculating, are the numbers
   a) $234567234123 \cdot 3$ divisible by 3?

   b) $234567234123 \cdot 33$ divisible by 3?

   c) $234567234123 \cdot 33$ divisible by 11?

2. Is number a divisible by number b? if yes, find the quotient.

   1) $a = 2 \cdot 2 \cdot 3 \cdot 7 \cdot 7,$ $b = 2 \cdot 2 \cdot 11$
   2) $a = 2 \cdot 3 \cdot 5 \cdot 13,$ $b = 5 \cdot 13$
   3) $a = 3 \cdot 5 \cdot 5 \cdot 11 \cdot 17,$ $b = 3 \cdot 5 \cdot 17$
   4) $a = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 19 \cdot 23,$ $b = 2 \cdot 2 \cdot 3 \cdot 5$
   5) $a = 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 11 \cdot 13,$ $b = 405$
   6) $a = 2 \cdot 3 \cdot 7 \cdot 11 \cdot 13 \cdot 29,$ $b = 2002$

4. Find GCF (with or without using prime factorization)
      $a.\ GCF(8, 48);$ $b.\ GCF(35,105);$ $c.\ GCF(46, 69);$

5. Find the LCM using the prime decomposition:
          $a.\ LCM(28, 35);$ $b.\ LCM(90, 96);$ $c.\ LCM(72, 90, 96);$

6. There are less than 100 apples in a box. They can be evenly divided between 2, 3, 4, 5, and 6 kids. How many apples are there in the box?

7. Two buses leave from the same bus station following two different routes. For the first bus, it takes 48 minutes to complete the roundtrip route. For the second one, it takes 1 hour and 12 minutes to complete the round trip route. How much time will it take for the buses to meet at the bus station for the first time after they have departed for their routes at the same time?

8. Boxes that are 30 cm tall are being piled next to boxes that are 40 cm tall. What is the least height at which the two piles will be the same height?

9. Numbers 100 and 90 were divided by the same number. In the case of 100, the remainder is 4, in the case of 90, the remainder is 18. What is the divisor?

10. Find missing digits in the problems:

```
   3 5 □ 7 8            5 □ 7 2 8
 + 4 □ 5 9 6          +   7 0 4 5
     6 7 8 □            8 3 □ 5 0
 ─────────────        ─────────────
   8 9 4 □ 5            8 2 1 □ □
                      ─────────────
                       2 2 7 1 6 5
```