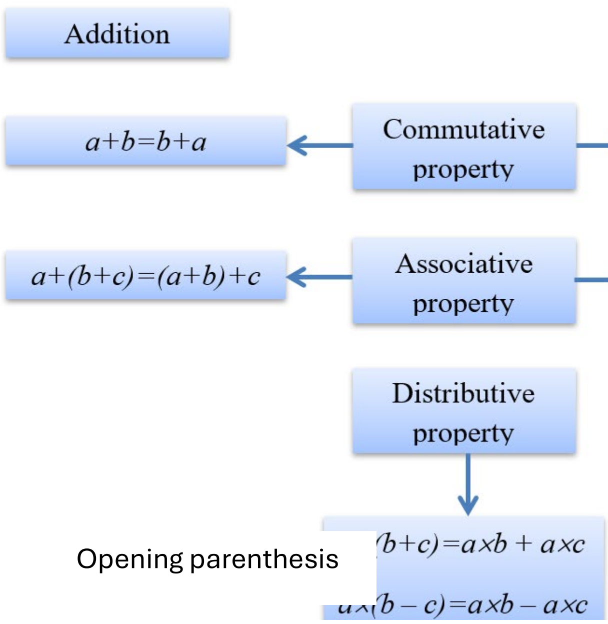
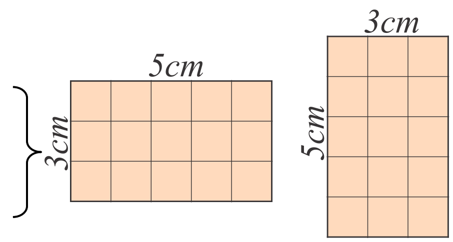
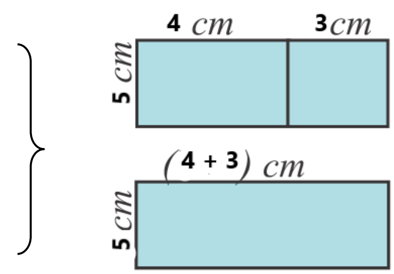
Math 4. Class Work 4

Distributive property, factorizing a common factor





going from right to left = factorizing a common factor.



Example (factorizing a common factor**):** The combined area of these 2 rectangles is $S = \boldsymbol{a} \times b + \boldsymbol{a} \times c$, but the rectangle with one side *a* cm and the other (*b*+*c*) cm will have exactly the same area. $S = \boldsymbol{a} \times (b + c)$.

Prime factorization.

Natural numbers greater than 1 that have no divisors other than 1 and themselves, are called **prime numbers**.

In mathematics, factorization is a decomposition of a number or mathematical expression as a product of numbers or/and expressions.

A number can be represented as the product of two or more other numbers, for example:

$40 = 4 \cdot 10 = 4 \cdot 2 \cdot 5 = 2 \cdot 2 \cdot 2 \cdot 5$ (the last products represent the prime factorization)

Prime numbers: 2, 3, 5, 7, 11, 13, ….. 2 is the only even prime number

Prime factorization is the decomposition of a natural number into the product of prime numbers. Any natural number has a single unique prime factorization.

Prime factors of 168 are $2, 2, 2, 3, 7$

Prime factorization process:

168	2
84	2
42	2
21	3
7	7
1	

The Greatest Common Factor or Divisor (GCF) of two numbers, is the largest number that can be a divisor for both numbers.

Example: $\text{GCF}(168, 180) = 12$

Find the prime factorization for both numbers, find the multiple common in both numbers

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168$$

$$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$$

$$\Rightarrow \; 2 \cdot 2 \cdot 3 = 12$$

The Least Common Multiple (LCM) of two numbers is the smallest number that is divisible by both of these numbers.

Example: $\text{LCM}(12, 15) = ?$ (smallest number divisible by both 12 and 15)

Find the prime factorization of 12 and 15:

12	2
6	2
3	3
1	

15	3
5	5
1	

$$\Rightarrow \quad 2 \cdot 2 \cdot 3 = 12$$

$$3 \cdot 5 = 15$$

$$\Rightarrow \quad 2 \cdot 2 \cdot 3 \cdot 5 = 60$$

The number we are looking for has to be a product of prime factors 12 and 15. As 3 is common, the LCM is a product of all factors in both without repeat, this number is 60.

Problems:

1. Without calculating, establish whether the sum is divisible by a number. Find a common factor and take it in front of the parenthesis.

$a.\ 25 + 35 + 15 + 45 \quad by\ 5;$ $\qquad b.\ 14 + 21 + 63 - 11 \quad by\ 7$

2. Factorize the common factor

Example: $3 \times 5 + 3 \times 7 = 3 \times (5 + 7)$

   a. $2 \times 3 + 2 \times 5 =$
   b. $3x + 3y =$
   c. $5a + 5b + 5c =$

3. Can any natural number be expressed as a product of 2 or more numbers other than 1 and itself? Find the product of the smallest factors.

$a. 11; \qquad b. 36; \qquad c. 40$

4. Even numbers are divisible by 2 (they have 2 as a divisor), so they can be factorized as 2 times something else. Is there any even prime number?

5. Prove that the following numbers are not prime:

$a.\ 25; \qquad b.\ 8192; \qquad c. 99.$

6. Find the prime factors of 168 and 180 using the prime factorization procedure

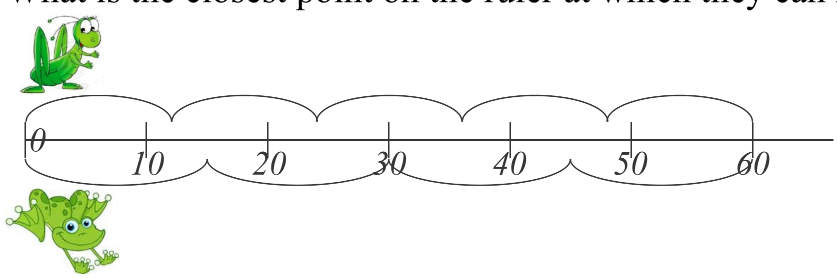
7. For Halloween, the Jonson family bought 168 mini chocolate bars and 180 gummy worms. What **is the largest number** of kids among whom the Jonsons can evenly divide both kinds of candy?

Procedure for GCF(168.180)

8. Find GCF (Greatest Common Factor) using prime decomposition:

$a. GCF(28, 35);$ $b.\ GCF(16, 56);$ $c. GCF(20, 100);$

9. A grasshopper jumps a distance of 12 centimeters each leap, while a little frog covers a distance of 15 centimeters per leap. They both start at 0 and hop along the long ruler. What is the closest point on the ruler at which they can meet?



There are specific points on the ruler that both of them can reach after a certain number of leaps. One of these points is, of course, $12 \times 15 = 180$ cm. A grasshopper would make 15 jumps, while a frog would make only 12.

Are there other smaller possible meeting points—other common multiples of 12 and 15? We are looking for the LCM—the smallest number divisible by both 12 and 15.

Procedure for LCM(12,15)

If time:

10. Find GCF (with or without using prime factorization)

$a.\ GCF(8, 48);$ $b.\ GCF(35,105);$ $c. GCF(46, 69);$

11. Find the LCM using the prime decomposition:

$a.\ LCM(18, 62);$ $b.\ LCM(264, 300);$ $c.\ LCM(72, 90, 96);$