

Exponent.

Exponentiation is a mathematical operation, written as a^n , involving two numbers, the base a and the exponent n . When n is a positive integer, exponentiation corresponds to repeated multiplication of the base: that is, a^n is the product of multiplying n bases:

$$a^n = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n \text{ times}}$$

In that case, a^n is called the n -th power of a , or a raised to the power n .

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The exponent indicates how many copies of the base are multiplied together. For example, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$. The base 3 appears 5 times in the repeated multiplication, because the exponent is 5. Here, 3 is the *base*, 5 is the *exponent*, and 243 is the *power* or, more specifically, *the fifth power of 3*, *3 raised to the fifth power*, or *3 to the power of 5*.

Properties of exponent:

$$a^{10} \cdot a^{15} = \underbrace{a \cdot a \dots \cdot a}_{10 \text{ times}} \cdot \underbrace{a \cdot a \dots \cdot a}_{15 \text{ times}} = \underbrace{a \cdot a \cdot a \dots \cdot a}_{10+15 \text{ times}} = a^{10+15} = a^{35}$$

$$(a^{10})^{15} = \underbrace{a^{10} \cdot a^{10} \cdot \dots \cdot a^{10}}_{15 \text{ times}} = \underbrace{a \cdot a \cdot \dots \cdot a}_{10 \text{ times}} \cdot \dots \cdot \underbrace{a \cdot a \cdot \dots \cdot a}_{10 \text{ times}} = a^{10 \cdot 15} = a^{150}$$

$$a^{10} \cdot a = \underbrace{a \cdot a \cdot a \dots \cdot a}_{10 \text{ times}} \cdot a = \underbrace{a \cdot a \cdot a \cdot a \dots \cdot a}_{10+1 \text{ times}} = a^{10+1} = a^{10} \cdot a^1$$

In order to have the set of power properties consistent, $a^1 = a$ for any number a .

$$a^{10} = a^{10} \cdot 1 = a^{10+0} = a^{10} \cdot a^0$$

In order to have the set of properties of exponent consistent, $a^0 = 1$ for any number a , but 0.

Also, if there are two numbers a and b :

$$(a \cdot b)^{10} = \underbrace{(a \cdot b) \cdot \dots \cdot (a \cdot b)}_{10 \text{ times}} = \underbrace{a \cdot \dots \cdot a}_{10 \text{ times}} \cdot \underbrace{b \cdot \dots \cdot b}_{10 \text{ times}} = a^{10} \cdot b^{10}$$

1. $a^n = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n \text{ times}}$

2. $a^n \cdot a^m = a^{n+m}$

3. $(a^n)^m = a^{n \cdot m}$

4. $a^1 = a$, for any a

5. $a^0 = 1$, for any $a \neq 0$

6. $(a \cdot b)^n = a^n \cdot b^n$

- A positive number raised into any power will result a positive number.
- A negative number, raised in a power, represented by an even number is positive, represented by an odd number is negative.

Exponent is very interesting mathematical operation. There is the story of the invention of the game of chess. The king ordered a new game because he was bored by the old games, was so happy about the new chess game that he said to the inventor: "*Name your reward and you will get it!*" The inventor asked for a simple reward. "*I would like to have one grain of rice on the first chess square, two on the second, four on the third and so on, doubling the amount of rice every square.*" The legend says that the King was surprised he didn't ask for gold but was quite content that the inventor asked for so little. But when the court scholars told him there wasn't enough rice in the whole world to fill the chess board, he had to admit his loss:

$$1 + 2 + 2^2 + 2^4 + \dots + 2^{63} = 18,446,744,073,709,551,615$$

The weight of the rice grain is about 0.03g. so:

$$1.8 \cdot 10^{19} \cdot 0.03 = 5.4 \cdot 10^{17} \text{ g. or about } 5.4 \cdot 10^{14} \text{ kg or } 10^{15} \text{ lb.}$$

Homework:

1. Evaluate:

$$a. 2^5; \quad b. (-2)^5; \quad c. -2^5; \quad d. (4 \cdot 2)^3; \quad e. \left(\frac{1}{6}\right)^2$$

2. What digits should be put instead of * to get true equality? How many solutions does each problem have?

$$a. (2 *)^2 = ** 1; \quad b. (3 *)^2 = *** 6 \quad c. (7 *)^2 = *** 5$$

3. A toy making company had a huge success launching their new plushies. At the end of the first month they sold a 1000 of them. Each next month their sales were growing by a factor of two (double). How many plushies in total they sold after 3 month?

4. Houses of Winnie the Pooh and Piglet are on the same street, 600 meters apart. At the same time, they started moving in opposite directions. Pooh was walking with a speed of 3 km/h, and Piglet was running with a speed of 6 km/h. How far from each other will they be in 20 minutes?

5. The cats of the Siberian, Angora, Persian, and Siamese breeds were presented at the cat exhibition. Siamese cats were twice as many as Angora cats, Persian cats were 1.5 times more than Siamese cats, and Siberian cats were 13 fewer than Persian cats. How many cats of each breed were there if there were 77 cats in total?