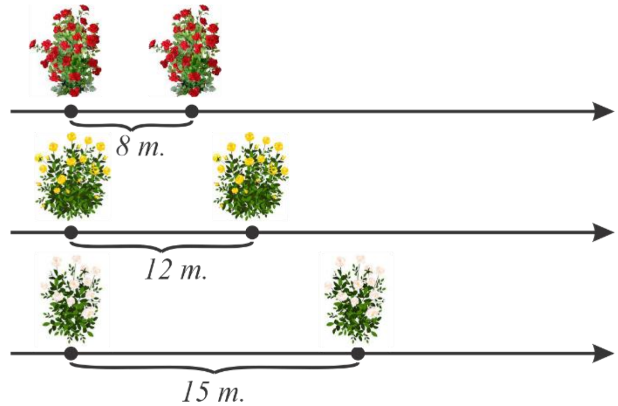
28. In a garden, there are red, yellow, and white rose bushes. At the beginning of the garden, red, yellow, and white bushes are planted in a row. Then, red roses are planted every 8 meters, yellow roses every 12 meters, and white roses every 15 meters, continuing until all three types of bushes are again planted in a row at the end of the garden. How long is this garden, and how many red, yellow, and white roses are planted there?



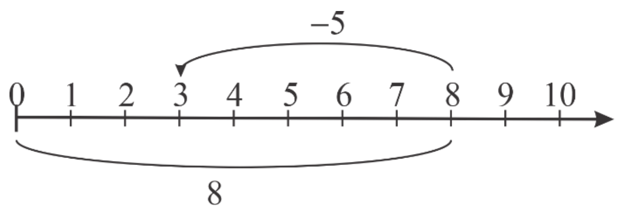
Chapter 7.

Positive and negative numbers.

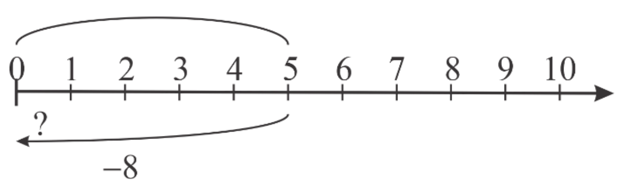
We have already observed that not every natural number can be divided by any other natural number. There are prime numbers, which can only be divided by 1 or by themselves. Other numbers are not prime, and are divisible by some numbers but not by others. To handle this we introduce a new kind of numbers – fractions – which are not natural numbers, but represent the parts of a whole.

What can we say about the operation of subtraction? Can we subtract any natural number from any other natural number? It's easy to subtract 5 from 8, but what happens if we need to subtract 8 from 5? Let's consider one example:

John has 8 dollars, and he wants to buy a 3-subject notebook, that costs 5 dollars. How much money will he have left?

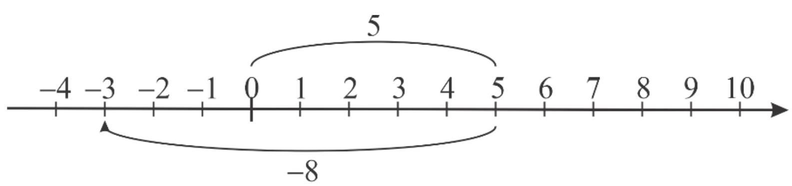


Now, what if he only has 5 dollars, but he needs to buy a 5-subject notebook, priced 8 dollars?

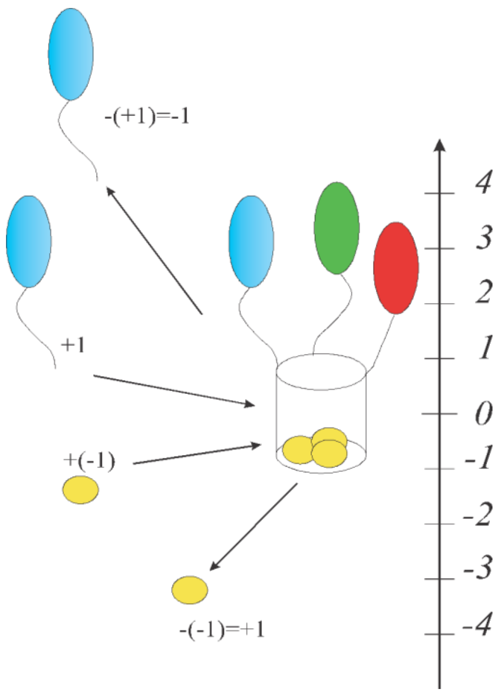


In this case, he can borrow 3 dollars from his friend, resulting in a balance of (-3) dollars (and a notebook).

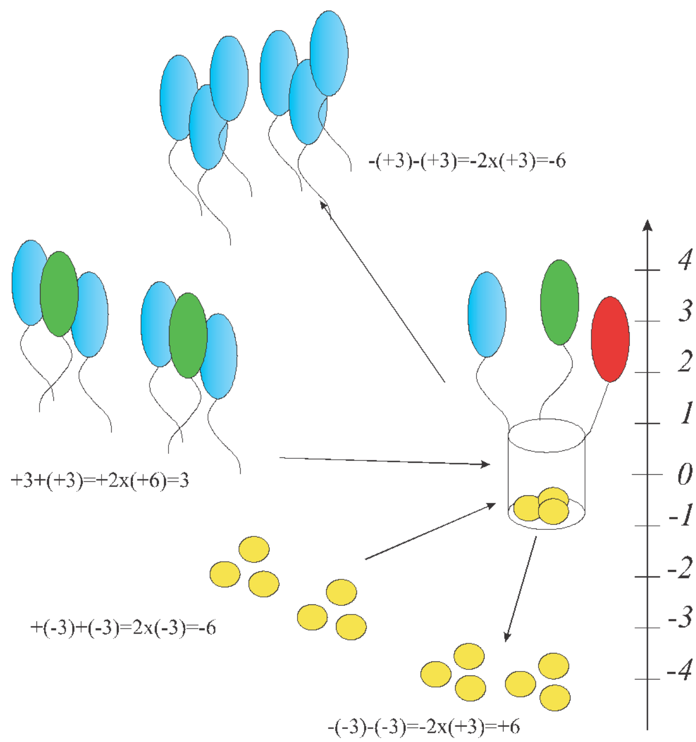
Another example of a negative numbers in our everyday life:



Temperature can be measured in different scales. In the USA we use Fahrenheit, in many other countries the Celsius scale is adopted. It based on a two reference points, water freezing temperature at 0°C and water boiling temperature at 100°C, 1°C is a 100th part of the 100°. The Fahrenheit scale was developed based on three fixed points of temperature. Originally, Fahrenheit chose the freezing temperature of a mixture of ice, water, and ammonium chloride as 0°F, the freezing point of water as 32°F, and the average human body temperature as 96°F. The scale was later adjusted so that water boils at 212°F, and the average human body temperature was redefined as approximately 98.6°F.



Negative numbers represent opposites. If "positive" represents movement to the right, "negative" represents movement to the left. If "positive" represents above sea level, then "negative" represents below sea level. If "positive" represents a deposit, "negative" represents a withdrawal. They are often used to represent the magnitude of a loss or deficiency. Negative numbers appeared for the first time in history in the Nine Chapters on the Mathematical Art, which in its present form dates from the period of the Chinese Han Dynasty (200 BCE – 220 CE 200), but may well contain much older material. Liu Hui (c. 3rd century) established rules for adding and subtracting negative numbers. By the 7th century, Indian mathematicians such as Brahmagupta were describing the use of negative numbers. Islamic mathematicians further developed the rules of subtracting and multiplying negative numbers and solved problems with negative coefficients. Western mathematicians accepted the idea of negative numbers by the 17th century. Prior to the concept of negative numbers, mathematicians such as Diophantus considered negative solutions to problems "false" and equations requiring negative solutions were described as absurd.



We can use the very clear illustration of how positive and negative numbers work. At the beginning, the basket has a few balloons attached and the same number of sand bags (weights) placed inside, is positioned at 0 and doesn't move. Balloons represent positive units; sand bags represent negative units. If we add one balloon, the basket gets pulled up by one unit. If we add one sand bag, the basket will be pulled down by one unit. If we remove one balloon, the basket will go one unit down, which is equivalent of adding one sand bag. So $-(+1) = +(-1)$. If we remove one sand bag, the basket will go one unit up, which is equivalent of adding one balloon. So $-(-1) = +(+1)$.

Two numbers that have the same magnitude but are opposite in signs are called opposite numbers.

Multiplication and division of negative numbers.
If we multiply 2 positive numbers, we will get a third positive number. What will happen if we multiply one negative and one positive number. Let's again review our model. In this case, we will add or remove our balloons and sand bags by groups of three. Addition of two groups of 3 sand bags will drive the basket 6 units down, because we add 6 bags. So $2 \times (-3) = -6$.
Removing of 2 groups of 3 sand bags will drive our basket 6 units up, which is corresponding of adding 6 balloons, so $-2 \cdot (-3) = 6$
Addition of 6 balloons (2 groups of three balloons) of cause will help us to move our basket up for 6 units. If we remove 2 groups of 3 balloons, we will descend 6 units.

$$-2 \cdot (+3) = -6$$

When we write a positive number, like (+6), we are usually omitting "+" sign, (+6) is the same as just 6. The expression $(+2) + (+6)$ means that we are adding two positive numbers, 2 and 6.

$$(+2) + (+6) = 2 + 6$$

When we write negative numbers, we have to be more careful; if the negative number is the first in the expression, it can be written as is:

$$(-2) + 6 = -2 + 6$$

If the negative number is not the first, it's a good idea to put it into parenthesis:
$6 + (-2)$. Expression written as $6 + -2$ looks awkward. Also, we know that the addition of a negative number is the same as subtraction of the positive one, opposite to negative $6 + (-2) = 6 - 2$.
If we want to subtract a positive number, we can write $6 - (+2)$, but the plus sign can be omitted: $6 - 2$. If we want to subtract a negative number, we have to add the number that is positive and opposite to negative.

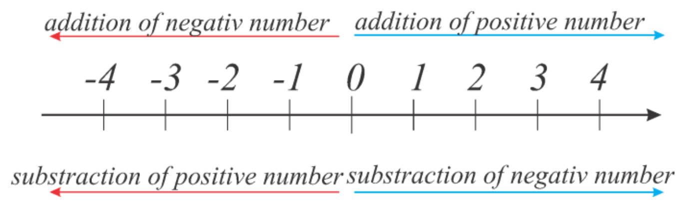
$$6 - (-2) = 6 + 2$$

Two numbers which sum is 0, are called **opposite** numbers.

6 and -6 are opposite numbers, $6 + (-6) = 0$. If we write "minus" sign in front of a number, we will get an opposite number:

$$-(-5) = 5; \qquad 5 + (-5) = 0$$

Natural numbers, their opposites, and 0 are called whole numbers or integers.



Exercises:
1. 2Write all integers

$a. from\ (-5) to\ 5;$ $\qquad b.\ from\ (-4) to\ 7;$ $\qquad c. from (-6)\ to\ 0.$

2. Write all negative whole numbers which are
   a. Greater than -8;
   b. Greater than -12 but smaller than -9;
   c. Smaller than 3 but greater than -11;

3. Fill up the table:

a	7	-4			5		0	
$-a$			0	-1		8		-3

4. Write a number, equal to

   Example: $-(-3) = 3$

   $a.\ -(+5);$ $b.\ +(+5);$ $c.\ -(+5);$

   d. $-(-(+5));$ $e.\ -(-(-5));$ $f.\ -\left(-\left(-(-5)\right)\right)$

   $g.\ \underbrace{-\left(-\left(-(\dots(+5))\right)\right)}_{10\ "-"signs};$ $h.\ \underbrace{\left(-\left(-(\dots(-5))\right)\right)}_{10\ "-"signs};$

   $i.\ \underbrace{-\left(-\left(-(\dots(+5))\right)\right)}_{11\ "-"signs}$

5. Evaluate:

   $a. -1\cdot 5;$ $b.\ 33\cdot(-1);$ $c. -101\cdot 0;$

   $d. -15\cdot(-1);$ $e.\ 0\cdot(-111);$ $f. -1\cdot 0;$

6. Evaluate:

   $a.\ (+6)\cdot(-4);$ $b.\ (+15)\cdot(-2);$ $c.\ (-12)\cdot 3;$

   $d.\ (-10):(+2);$ $e.\ (15):(-3);$ $f.\ (-1)\cdot(+1001);$

7. Evaluate:

   $a.\ (-8)\cdot(-3);$ $b.\ (-11)\cdot(-4);$ $c.\ (-5)\cdot(-10);$

   $d.\ (-15):(-3);$ $e.\ (-12):(-4);$ $f.\ \left(-\frac{1}{5}\right)\cdot(-10);$

8. Find the unknown factor:

   $a.\ 345\cdot x = -345;$ $b.\ a\cdot(-512) = -512;$

   $c.\ x\cdot(-178) = 0;$ $d. -421\cdot b = 421$

9. Find the unknown factor

   $a. -10\cdot x = 70;$ $b.\ -4\cdot y = -16;$

   $c.\ x\cdot(-12) = 36;$ $d.\ y\cdot(-3) = -21$

10. Fill the table:

a	-1	4	10	-8	-4
b	1	-2	2	5	-3
c	3	-6	-5	-6	-2
$a \cdot b \cdot c$					
$(-a) \cdot b \cdot c$					
$(-a) \cdot (-b) \cdot c$					
$(-a) \cdot (-b) \cdot (-c)$					

11. Positive number, negative number or 0 is the result of the products:

$a.$ $(-1) \cdot (-2) \cdot (-3) \cdot (-4) \cdot (-5) \cdot (-6) \cdot (-7) \cdot (-8) \cdot (-9);$

$b.$ $(-1) \cdot (-2) \cdot (-3) \cdot (-4) \cdot (-5) \cdot (-6) \cdot (-7) \cdot (-8);$

$c.$ $(-3) \cdot (-2) \cdot (-1) \cdot 0 \cdot 1 \cdot 2 \cdot 3;$

$d.$ $1 \cdot (-1) \cdot 2 \cdot (-2) \cdot 3 \cdot (-3) \cdot \ldots \cdot 10 \cdot (-10);$

12. Evaluate:

$a.$ $-20 : 4 - 9;$ $b.$ $5 + 11 \cdot (-2);$ $c.$ $(-7 + 5) : (-1);$

$d.$ $-27 : (-3) - 10;$ $e.$ $0 - (-28) : (-7);$ $c.$ $-36 : (-8 + 20);$

13. Rewrite without parenthesis:

Example:

$$30 - (2 - 1) = 30 - 2 + 1$$

To check your solution, you can find the value for both part of the equality:

$30 - (2 - 1) = 30 - 2 + 1;$

$$30 - (2 - 1) = 30 - 1 = 29, \quad 30 - 2 + 1 = 29$$

$a.$ $20 + (2 - 3);$ $b.$ $20 - (2 - 3);$

$c.$ $20 - (-2 + 3);$ $d.$ $20 - \big(-2 + (-3)\big);$

14. Compare:

$-4 \ldots 4$ $6 \ldots -4$ $\frac{2}{3} \ldots -\frac{3}{2}$

$-4 \ldots -2$ $-4 \ldots 0$ $-\frac{2}{3} \ldots -1$

$-4 \ldots -6$ $-1 \ldots -\frac{1}{2}$ $-2 \ldots \frac{1}{2}$

15. Evaluate:

$3 + (-2);$ $3 + (+2);$ $-3 - (-2);$

$3 - (+2);$ $-3 + (-2);$ $-3 + (+2);$

$3 - (-2);$ $-3 - (+2);$ $-3 + (+3);$

16. Compare without doing actual calculation.

$a.\ \ 100 - (35 - 20) \quad 100 - (35 + 20)$

$b.\ \ 100 + (35 - 20) \quad 100 + (35 + 20)$

$c.\ \ 100 - (-35 - 20) \quad 100 - (-35 + 20)$

$d.\ \ 100 + (-35 - 20) \quad 100 + (-35 + 20)$

17. Positive or negative number will be the product of
   a. Two negative and one positive numbers.
   b. One negative and two positive numbers
   c. Three negative numbers.
   d. Three positive numbers.

18. Some digits in the numbers were changed for asterisks. Can you compare numbers?

$a.\ \ ** \ldots (-1**);$ $b. -8** \ldots -9**;$ $c. -7**8* \ldots 7**2*$

19. Using the distributive property, represent the numerator of each fraction as a product and reduce the fractions:

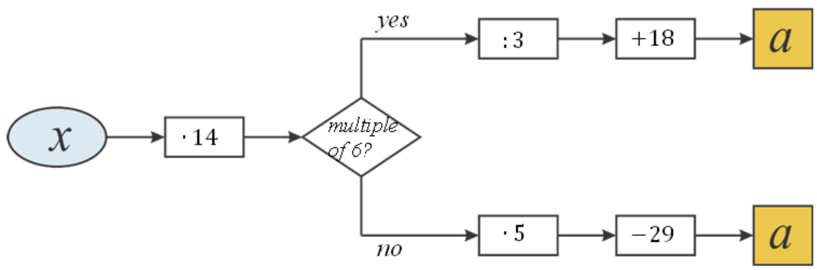
Example:

$$\frac{32\cdot 5 + 32\cdot 9}{160\cdot 28} = \frac{32\cdot(5+9)}{1602\cdot 28} = \frac{32\cdot 14}{160\cdot 28} = \frac{32\cdot 14}{32\cdot 5\cdot 14\cdot 2} = \frac{1}{5\cdot 2} = \frac{1}{10}$$

$$a.\ \frac{15\cdot 9 - 15\cdot 6}{9\cdot 30}; \qquad b.\ \frac{17\cdot 4 + 17\cdot 9}{34\cdot 52};$$

$$c.\ \frac{18\cdot 7 + 18\cdot 3}{1200}; \qquad d.\ \frac{24\cdot 11 - 24\cdot 3}{300}$$

20. Using the algorithm fill the table:



x	1	2	3	4	5	6	7	8	9	10
a										

21. Victor had 80 dollars more than Mary. After Victor spent half of his money, he had 10 dollars less than Mary. How much money did Mary and Victor have together initially?

22. A problem from the "Arithmetic" of the famous Central Asian mathematician Muhammad ibn Musa al-Khwarizmi (9th century AD).
"Find the number, knowing that if you subtract one-third and one-fourth (of this number) from it, you get 10."

23. What sign should replace the asterisks, and what digit should be filled in to make the following expressions true equalities?
$a.\ 87 * 29 = 5\ ;$ $b.\ 2:4 * 77 = 1\ ;$ $c.\ 18 * 3 \cdot 9 =\ 5;$
$d.\ (96 * 48):8 = ;$ $e.\ 3 * 5 - 73 = 14\ ;$ $f.\ 300 - (80 * 3) * 6 = 2\ \ 0;$

24. Five friends were sledding down a hill. John went farther than Mike, but not as far as Robert. Dennis went a shorter distance than Mike, while Peter went farther than Robert. Who among the boys went the farthest, and who went the shortest distance?

25. Find the number, if
$a.\ \frac{2}{3}\ of\ this\ number\ is\ 6;$ $b.\ \frac{2}{5}\ of\ this\ number\ is\ 22;$ $c.\ \frac{3}{7}\ of\ this\ number\ is\ 12;$

26. Find
$a.\ \frac{2}{3}\ of\ number\ 6;$ $b.\ \frac{2}{5}\ of\ number\ is\ 25;$ $c.\ \frac{3}{7}\ of\ number\ is\ 49;$

27. Draw a number line in your notebook. Mark the fractions $\frac{1}{18}$; $\frac{1}{9}$; $\frac{1}{6}$; $\frac{1}{3}$; $\frac{1}{2}$. How many cells the unit segment of the scale should be to conveniently mark all these fractions on a same number line?

28. Compare fractions: