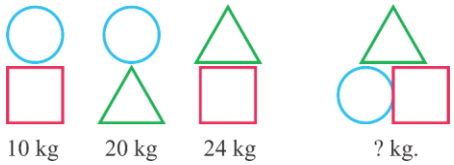
31. Find the weight of each shape:




## Chapter 6.

**Complex fractions.**

Complex fractions are formed by two fractional and/or numeral expressions, one on the top and the other one at the bottom, for example:

$$\frac{(2+3)\cdot 5}{7-\frac{1}{2}}; \qquad \frac{\frac{1}{2}+\frac{1}{3}}{\frac{7}{9}-\frac{2}{5}}$$

We know that the fraction bar is a just another way to write the division sign, so, the above expressions are equivalent to

$$\frac{(2+3)\cdot 5}{7-\frac{1}{2}} = ((2+3)\cdot 5):\left(7-\frac{1}{2}\right); \qquad \frac{\frac{1}{2}+\frac{1}{3}}{\frac{2}{3}+\frac{1}{4}} = \left(\frac{1}{2}+\frac{1}{3}\right):\left(\frac{2}{3}+\frac{1}{4}\right)$$

It is easy to simplify a complex fraction:

$$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{2}{3}+\frac{1}{4}} = \left(\frac{1}{2}+\frac{1}{3}\right):\left(\frac{2}{3}+\frac{1}{4}\right) = \frac{\frac{3}{6}+\frac{2}{6}}{\frac{8}{12}+\frac{3}{12}} = \frac{\frac{5}{6}}{\frac{11}{12}} = \frac{5}{6}:\frac{11}{12} = \frac{5}{6}\cdot\frac{12}{11} = \frac{5}{1}\cdot\frac{2}{11} = \frac{10}{11}$$

Now let's simplify a little more sophisticated complex fraction:

Example 1.

$$\frac{1\frac{2}{3}+2\frac{4}{9}}{4\frac{26}{27}-2\frac{2}{9}} = \left(1\frac{2}{3}+2\frac{4}{9}\right):\left(4\frac{26}{27}-2\frac{2}{9}\right)$$

$$1\frac{2}{3}+2\frac{4}{9} = 1+\frac{2}{3}+2+\frac{4}{9} = 3+\frac{6}{9}+\frac{4}{9} = 3+\frac{10}{9} = \frac{27}{9}+\frac{10}{9} = \frac{37}{9}$$

$$4\frac{26}{27}-2\frac{2}{9} = 4+\frac{26}{27}-2-\frac{2}{9} = 2+\frac{26}{27}-\frac{6}{27} = 2+\frac{20}{27} = \frac{54}{27}+\frac{20}{27} = \frac{74}{27}$$

$$\frac{37}{9}:\frac{74}{27} = \frac{37}{9}\cdot\frac{27}{74} = \frac{37\cdot 3\cdot 9}{9\cdot 2\cdot 37} = \frac{3}{2} = 1\frac{1}{2}$$

Example 2:

$$\frac{3\frac{4}{7}:\left(6\frac{1}{28}-3\frac{3}{4}\right)}{\left(1\frac{5}{6}\cdot 1\frac{5}{22}\right):18\cdot 5}$$

First, let's find the value of the numerator:

$$3\frac{4}{7}:\left(6\frac{1}{28}-3\frac{3}{4}\right)=\frac{25}{7}:\left(5+1\frac{1}{28}-3-\frac{3}{4}\right)=\frac{25}{7}:\left(2+\frac{29}{28}-\frac{3}{4}\right)==\frac{25}{7}:\left(2+\frac{29}{28}-\frac{21}{28}\right)=\frac{25}{7}:\left(2\frac{8}{28}\right)$$
$$=\frac{25}{7}:\frac{64}{28}=\frac{25}{7}\cdot\frac{28}{64}=\frac{25\cdot 4}{64}=\frac{25}{16}$$

The value of the denominator is

$$\left(1\frac{5}{6}\cdot 1\frac{5}{22}\right):18\cdot 5=\left(\frac{11}{6}\cdot\frac{27}{22}\right)\cdot\frac{1}{18}\cdot 5=\frac{11\cdot 3\cdot 9}{2\cdot 3\cdot 11\cdot 2}\cdot\frac{1}{2\cdot 9}\cdot 5=\frac{5}{8}$$

Finally:

$$\frac{\frac{25}{16}}{\frac{5}{8}}=\frac{25}{16}:\frac{5}{8}=\frac{25}{16}\cdot\frac{8}{5}=\frac{5\cdot 5\cdot 8}{2\cdot 8\cdot 5}=\frac{5}{2}=2\frac{1}{2}$$

**Problem solving examples:**

*What number $x$ can be substituted with so that the fraction $\frac{x}{18}$ will be a nonreducible proper fraction?*

For the fraction $\frac{x}{18}$ to be proper, $x$ should be less than 18, and greater than 0. We have numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17. All of them greater than 0 and less than 18. For the fraction to be nonreducible, the numerator and denominator should not have common factors. 18 can be prime factorized as:

$$18=2\cdot 3\cdot 3$$

So, we have to exclude all even numbers, 2, 4, 6, 8, 10, 12, 14, 16.
The numbers left are 1, 3, 5, 7, 9, 11, 13, 15, 17.
Then we have to exclude all numbers divisible by 3: 3, 9, 15. Numbers that are not divisible by 3 will not be divisible by 6 $(2\cdot 3)$ and 9 $(3\cdot 3)$ as well. So, $x$ can be substituted with 1, 5, 7, 11, 13, and 17. All these fractions are proper and nonreducible:

$$\frac{1}{18};\frac{5}{18};\frac{7}{18};\frac{11}{18};\frac{13}{18};\frac{17}{18};$$

*56 tickets were sold for the flight, and 24 seats remained unoccupied. What fraction of the seats are occupied?*

Step 1. Total number of seats is $56+24=80$
Step 2. One seat is $\frac{1}{80}$ part of the total number of seats. 56 such parts are $\frac{56}{80}$.

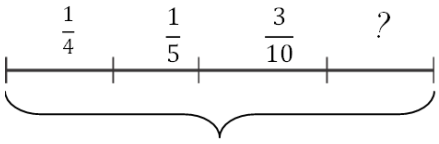
$$56:80=\frac{56}{80}=\frac{7\cdot 8}{10\cdot 8}=\frac{7}{10}$$

Answer: $\frac{7}{10}$ of all seats are occupied.

*A cyclist covered $\frac{1}{4}$ of the distance in the first hour. During the second hour he drove $\frac{1}{5}$ of the distance, and during the third hour he covered $\frac{3}{10}$ of the distance. Which part of the planned distance does he still need to cover?*

We need to add

$$\frac{1}{4}+\frac{1}{5}+\frac{3}{10};$$



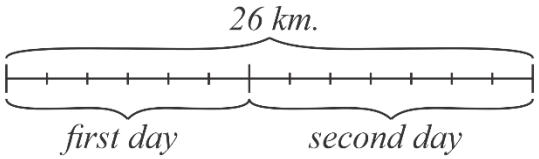

The common denominator is 10,

$$\frac{1}{4}+\frac{1}{5}+\frac{3}{10}=\frac{1\cdot 5}{4\cdot 5}+\frac{1\cdot 4}{4\cdot 5}+\frac{3\cdot 2}{10\cdot 2}=\frac{5+4+6}{20}=\frac{15}{20}=\frac{3}{4}$$

$$1-\frac{3}{4}=\frac{4}{4}-\frac{3}{4}=\frac{1}{4}$$

A backpacker walked 26 kilometers in 2 days. On the first day, he walked 6/7 of the distance he walked on the second day. How many kilometers did he walk each day?

On the first day, he walked exactly 1/7 less than on the second day. If we divide the distance he walked on the second day into 7 equal parts, we need to take 6 of those parts to determine how far he walked on the first day.




One seventh part of the second day's distance is

$$26:(6+7)=\frac{26}{6+7}=2\ km$$

On the first day a backpacker walked $2\cdot 6=12\ km$ and on the second day he walked $2\cdot 7=14\ km$.

***Exercises***:

1. Evaluate:

$$a.\frac{6}{1-\frac{1}{3}};\qquad b.\ \frac{1-\frac{1}{6}}{2+\frac{1}{6}};\qquad c.\ \frac{\frac{1}{2}+\frac{3}{4}}{\frac{1}{2}};\qquad d.\ \frac{\frac{7}{10}+\frac{1}{3}}{\frac{7}{10}+\frac{1}{2}};\qquad e.\frac{2-\frac{\frac{1}{2}-\frac{1}{4}}{2}}{2+\frac{\frac{1}{2}-\frac{1}{4}}{2}}$$

2. Write the expressions as fractions and evaluate:

$a.\ 14:42;\qquad b.\ 2:3:5;\qquad c.\ 2:8\cdot 3;\qquad d.\ 100\cdot 6:40;\qquad e.\ 5:15\cdot 3$

$f.\ (21\cdot 18):14;\qquad g.\ 50:(16\cdot 25);\qquad h.\ (12\cdot 15):40;\qquad i.\ (4\cdot 24):(2\cdot 8)$

3. Write the expressions as fractions and evaluate:

*a.* $(3 \cdot 3 \cdot 5 \cdot 11) : (3 \cdot 11)$; *b.* $(2 \cdot 2 \cdot 3 \cdot 5 \cdot 7) : (2 \cdot 3 \cdot 7)$;

*c.* $(2 \cdot 3 \cdot 7 \cdot 13) : (3 \cdot 7)$; *d.* $(3 \cdot 5 \cdot 11 \cdot 17 \cdot 23(: (3 \cdot 11 \cdot 17)$;

4. Evaluate:

$$\frac{3\frac{5}{11} \cdot 6\frac{3}{4}}{3\frac{5}{11} \cdot 6\frac{3}{4} + 3\frac{5}{11} \cdot 1\frac{1}{2}}$$

5. Into how many equal $\frac{1}{5}$ kilogram portions 5 kg cake can be divided?

6. The area of a rectangle is $\frac{5}{7}\ m^2$. One side of this rectangle is $\frac{3}{4}\ m$. What is the length of the other side?

7. Julia and Mary ate all the Halloween candy. Mary claimed that she ate $\frac{2}{3}$ of the candies, and Julia said that she ate $\frac{3}{5}$. Their parents think that something is wrong. Are they right?

8. The model of the house is $\frac{1}{25}$ of its real size. The width of a window on the model is 5 cm. How wide is a window in a real house?

9. What is the length of a segment if
   *a.* $\frac{2}{5}$ of its length is 12 meters;
   *b.* $\frac{3}{4}$ of its length is 9 centimeters;
   *c.* $\frac{3}{5}$ of its length is 15 millimeters.
   *d.* $\frac{2}{7}$ of its length is 8 meters.

10. What number $x$ can be substituted with so that the fraction $\frac{x}{12}$ will be a nonreducible proper fraction?

11. From 42 m of fabric, 10 identical duvet covers were sewn, and from 33 m - 15 identical sheets. How much fabric is needed for a set that includes 1 sheet and 1 duvet cover?

12. Mary's 10 steps are 9 meters, while Julia's 20 steps are 17 meters. Whose steps are longer?

13. The sum of all numbers in each square is 10. What number should be placed instead of "?" ?

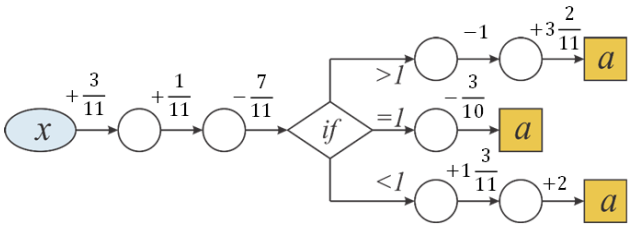
| | |
|---|---|
| $2\frac{1}{7}$ | $5\frac{4}{7}$ |
| $\frac{3}{7}$ | ? |

| | |
|---|---|
| $1\frac{4}{5}$ | $3\frac{2}{5}$ |
| ? | $2\frac{1}{5}$ |

| | |
|---|---|
| $\frac{5}{9}$ | ? |
| $2\frac{7}{9}$ | $1\frac{2}{9}$ |

| | |
|---|---|
| ? | $6\frac{8}{11}$ |
| $\frac{2}{11}$ | $2\frac{5}{11}$ |

14. Using the given algorithm, fill the table:




| $x$ | $\frac{6}{11}$ | $\frac{14}{11}$ | $\frac{4}{11}$ | $\frac{3}{11}$ | $1\frac{7}{11}$ |
|---|---|---|---|---|---|
| $a$ | | | | | |

15. Find the unknown:

$a.\ \frac{3}{5}+x=\frac{4}{5};$ $b.\ y-\frac{2}{7}=\frac{6}{7};$

$c.\ \frac{11}{25}-k=\frac{7}{25};$ $d.\ s+\frac{2}{9}=\frac{1}{9}+\frac{7}{9};$

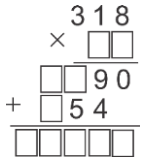
16. The weight of a turkey is three times less than the weight of a sheep, and the weight of three such sheep is 60 kg more than the weight of five turkeys. What is the weight of one turkey, and what is the weight of one sheep?

17. Solve the following problems:

a. How many different 2-digit numbers (excluding numbers with repeating digits, like 44) can be created using the digits 1, 2, 3, and 4?

b. How many teams of two students can be formed from 4 students: Mary, Elisabeth, John, and Mickle? What is similar, and what is different in these problems?

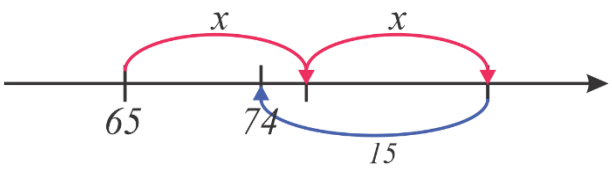
18. Fill in the empty spaces:




19. Without performing the multiplication, arrange the products in ascending order:

$56 \cdot 24;\quad 56 \cdot 49;\quad 13 \cdot 24;\quad 13 \cdot 11;\quad 74 \cdot 49;\quad 7 \cdot 11.$

20. Find $x$ on the picture below:



21. Write five values for $x$ such that:

$$\frac{1}{4} < x < \frac{1}{2}$$

22. Evaluate:

Example:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6};$$

One way to solve this is by bringing all fractions to a common denominator, 60:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} = 1 - \frac{30}{60} + \frac{20}{60} - \frac{15}{60} + \frac{12}{60} - \frac{10}{60} = 1 + \frac{32}{60} - \frac{55}{60} = \frac{60}{60} - \frac{55}{60} + \frac{32}{60}$$

$$= \frac{5}{60} + \frac{32}{60} = \frac{37}{60};$$

Another way to calculate it is to multiply the whole expression by the LCM of all denominators to bring everything to whole numbers:

$$\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right) \cdot 60 = 60 - 30 + 20 - 15 + 12 - 10 = 37$$

But the answer is now 60 times larger than the one we're looking for.

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} = 37 : 60 = \frac{37}{60}$$

$a.$ $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \frac{1}{18};$ $b.$ $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64}$

23. If 1 is added to both the numerator and denominator of an irreducible fraction, will it necessarily remain irreducible? Can it remain irreducible?

24. If 1 is added to both the numerator and denominator of a reducible fraction, can it remain reducible? Can it become irreducible?

25. Do the calculations with time units:

$a.$ $18min.36s. + 24min.58s.;$ $b.$ $5h.17min. - 3h.45min;$

$c.$ $42min.48s. : 24;$ $d.$ $3h.25min. \cdot 7;$

26. A giraffe is 12 times heavier than a kangaroo, and an elephant is 5 times heavier than the giraffe. What is the weight of each of these animals if their total weight is 5 tons 110 kg? ($1ton = 1000\,kg$.

27. *The inhabitants of the Unknown Planet, divide the day into several hours, an hour into several minutes, and a minute into several seconds. However, on their planet, there are 77 minutes in a day and 91 seconds in an hour. How many seconds are in a day on the Unknown Planet?