

Chapter 5.**Multiplication of a fraction by a number.**

To multiply a fraction by a number, simply multiply the numerator by the number:

$$\frac{2}{7} \cdot 3 = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{2+2+2}{7} = \frac{3 \cdot 2}{7} = \frac{6}{7}$$

On the other hand:

$$\frac{2}{7} \cdot 3 = 3 \cdot \frac{2}{7} = 3:7 \cdot 2 = (3:7) \cdot 2 = (3 \cdot 2):7 = \frac{3 \cdot 2}{7}$$

Multiplication of a fraction by a fraction.

$\frac{1}{15}$ is a part of a whole divided into 15 equal small parts.

If we want to take $\frac{1}{9}$ part of this little $\frac{1}{15}$ chunk, we have to divide it into 9 even smaller pieces, to find $\frac{1}{9}$ th of $\frac{1}{15}$ th.

$$\frac{1}{15} : 9 = \frac{1}{15} \cdot \frac{1}{9} = \frac{1}{15 \cdot 9} = \frac{1}{135}$$

If we need to take two small $\frac{1}{9}$ of $\frac{1}{15}$

$$\frac{1}{15} : 9 \cdot 2 = \frac{1}{15} \cdot \frac{2}{9} = \frac{1 \cdot 2}{15 \cdot 9} = \frac{2}{135}$$

Or we want to find out $\frac{2}{9}$ of $\frac{3}{15}$.

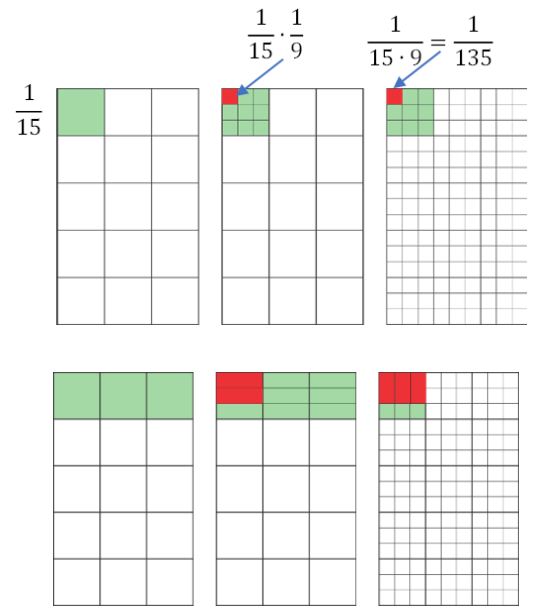
$$\frac{3}{15} : 9 \cdot 2 = \frac{3}{15} \cdot \frac{2}{9} = \frac{3 \cdot 2}{15 \cdot 9} = \frac{6}{135}$$

To multiply two fractions, we need to multiply numerators, multiply denominators and reduce fraction, if possible.

Examples:

$$\frac{3}{8} \cdot \frac{2}{7} = \frac{3 \cdot 2}{4 \cdot 2 \cdot 7} = \frac{3 \cdot 2}{4 \cdot 7 \cdot 2} = \frac{3}{4 \cdot 7} = \frac{3}{28}$$

Division of fractions.

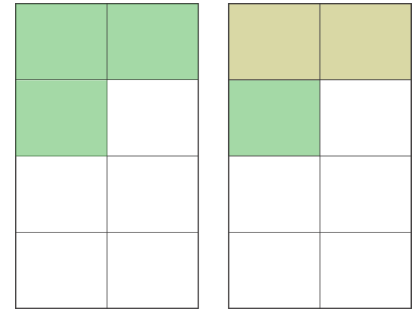


More of multiplication of fractions:

$$\frac{3}{8} \cdot \frac{2}{3} = \frac{2}{8} = \frac{1}{4}$$

By definition of the division operation, division of $\frac{1}{4}$ by $\frac{2}{3}$ should give the quotient $\frac{3}{8}$.

$$\frac{1}{4} : \frac{2}{3} = \frac{3}{8}$$



We can notice that the multiplication of $\frac{1}{4}$ by the inverse fraction $\frac{3}{2}$ will bring exactly $\frac{3}{8}$;

$$\frac{1}{4} \cdot \frac{3}{2} = \frac{1 \cdot 3}{4 \cdot 2} = \frac{3}{8}$$

To divide one fraction by another we need to multiply the dividend by the inverse fraction. Two fractions are inverse fractions if their product is 1. Inverse fractions can also be called reciprocal.

Two fractions are called *inverse fractions* if their product is equal to 1.

Examples:

$$\frac{1}{4} \cdot \frac{4}{1} = 1; \quad \frac{3}{5} \cdot \frac{5}{3} = 1; \quad \frac{4}{7} \cdot \frac{7}{4} = 1;$$

Exercise:

1. Evaluate:

$$\begin{array}{llll} a. \frac{3}{8} + 2\frac{1}{4}; & b. 2\frac{1}{3} - 1\frac{1}{2}; & c. \frac{1}{4} + 3\frac{1}{6}; & d. 4\frac{1}{5} - 2\frac{3}{10}; \\ e. 5\frac{5}{12} + 3\frac{2}{9}; & f. 7\frac{1}{9} - 4\frac{1}{3}; & g. 2\frac{4}{9} + \frac{1}{6}; & h. 2\frac{2}{7} - 1\frac{3}{5}; \\ i. 4\frac{3}{5} + 10\frac{1}{4}; & j. 6\frac{1}{4} - 3\frac{2}{5}; & & \end{array}$$

2. Evaluate:

Example:

$$4 - \frac{5}{6} = 3 + 1 - \frac{5}{6} = 3 + \frac{6}{6} - \frac{5}{6} = 3 + \frac{1}{6} = 3\frac{1}{6}$$

$$\begin{array}{llll} a. 1 - \frac{1}{3}; & b. 1 - \frac{11}{20}; & c. 4 - \frac{1}{9}; & d. 6 - \frac{3}{7}; \\ e. 1 - \frac{3}{4}; & f. 3 - \frac{1}{2}; & h. 5 - \frac{2}{5}; & i. 8 - \frac{2}{3}; \end{array}$$

3. Evaluate:

$$a. 5\frac{2}{6} - \frac{5}{6}; \quad b. 4\frac{5}{9} - \frac{8}{9}; \quad c. 6\frac{3}{7} - 5\frac{5}{7}; \quad d. 2\frac{1}{3} - \frac{2}{3};$$

$$e. 3\frac{1}{12} - 1\frac{5}{12}; \quad f. 4\frac{1}{8} - 3\frac{5}{8};$$

4. Write division operation as fraction, reduce fraction, if possible:

Example:

$$4:8 = \frac{4}{8} = \frac{1 \cdot 4}{2 \cdot 4} = \frac{1}{2}$$

$$a. 3:7; \quad b. 5:15; \quad c. 3:9; \quad d. 4:9; \quad e. 12:13.$$

5. Write fraction as a division, reduce fraction if possible:

Example:

$$\frac{3}{12} = 3:12 = \frac{1}{4} = 1:4$$

$$a. \frac{4}{5}; \quad b. \frac{7}{9}; \quad c. \frac{5}{15}; \quad d. \frac{2}{8}; \quad e. \frac{11}{44}.$$

6. Evaluate:

$$a. \frac{4}{5} \cdot \frac{5}{7}; \quad b. \frac{2}{3} \cdot \frac{5}{7}; \quad c. \frac{8}{9} \cdot \frac{3}{5}; \quad d. \frac{1}{4} \cdot \frac{1}{2}; \quad e. \frac{9}{2} \cdot \frac{2}{9};$$

$$f. \frac{8}{21} \cdot \frac{7}{10}; \quad g. \frac{3}{4} \cdot \frac{1}{2}; \quad h. \frac{8}{15} \cdot \frac{25}{28}; \quad i. \frac{5}{6} \cdot \frac{7}{12}; \quad j. \frac{4}{9} \cdot \frac{8}{9};$$

7. Painter painted $\frac{2}{7}$ of the house in 4 days. How many days will it take him to paint the whole house?

8. Evaluate:

$$a. \frac{3}{7} \cdot 2; \quad b. 3 \cdot \frac{1}{6}; \quad c. 9 \cdot \frac{5}{6}; \quad d. 2\frac{1}{3} \cdot 2; \quad e. 4 \cdot 1\frac{1}{2};$$

9. Evaluate:

$$a. \frac{1}{3} \cdot \frac{2}{7}; \quad b. \frac{1}{2} \cdot \frac{5}{6}; \quad c. \frac{1}{2} \cdot \frac{1}{3}; \quad d. \frac{3}{5} \cdot \frac{1}{2} \cdot \frac{4}{9}; \quad e. \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5}$$

$$f. \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdots \frac{23}{24} \cdot \frac{24}{25}$$

10. Evaluate:

Example:

$$3\frac{1}{4} \cdot 2 = \left(3 + \frac{1}{4}\right) \cdot 2 = 3 \cdot 2 + \frac{1}{4} \cdot 2 = 6 + \frac{1 \cdot 2}{4} = 6 + \frac{2}{4} = 6 + \frac{1}{2} = 6 + \frac{1}{2} = 6\frac{1}{2}$$

Or it can be done by transferring the mixed number into the improper fraction:

$$3\frac{1}{4} \cdot 2 = \frac{3 \cdot 4 + 1}{4} \cdot 2 = \frac{13}{4} \cdot 2 = \frac{13 \cdot 2}{4} = \frac{26}{4} = \frac{24 + 2}{4} = 6 + \frac{2}{4} = 6 + \frac{1}{2} = 6\frac{1}{2}$$

$$a. 2\frac{1}{3} \cdot 2; \quad b. 4 \cdot 1\frac{1}{2}; \quad c. 1\frac{1}{3} \cdot 9; \quad d. \frac{3}{7} \cdot 2\frac{1}{3}; \quad e. 1\frac{1}{3} \cdot 1\frac{2}{2}$$

$$f. 12 \cdot \frac{1}{6} \cdot 1\frac{1}{2} \cdot 3\frac{3}{4} \cdot 4\frac{1}{5}; \quad g. 3 \cdot 5\frac{1}{4} \cdot 1\frac{1}{7} \cdot 5\frac{1}{2} \cdot \frac{4}{11};$$

11. Write the fraction inverse to the given fraction:

Example:

$$\frac{3}{4} \rightarrow \frac{4}{3}; \quad \frac{3}{4} \cdot \frac{4}{3} = 1$$

$$a. \frac{3}{7}; \quad b. \frac{7}{9}; \quad c. \frac{12}{5}; \quad d. \frac{9}{4}; \quad e. \frac{1}{3}; \quad f. \frac{1}{3}; \quad g. \frac{m}{n} \quad (m, n \neq 0);$$

12. Evaluate:

$$a. \frac{2}{3} : \frac{5}{7}; \quad b. \frac{1}{4} : \frac{1}{2}; \quad c. \frac{3}{4} : \frac{1}{2}; \quad d. \frac{4}{9} : \frac{8}{9}; \quad e. \frac{5}{6} : \frac{7}{12};$$

$$f. 2 : \frac{1}{7}; \quad g. 4 : \frac{2}{3}; \quad h. 3 : \frac{1}{2}; \quad i. 1 : \frac{2}{7}; \quad j. 1 : \frac{1}{4};$$

13. Evaluate:

$$a. \frac{4}{7} \cdot \frac{5}{24} : 1\frac{1}{14}; \quad b. 25 \cdot \frac{7}{15} : \frac{7}{9}; \quad c. \frac{7}{18} : \frac{20}{21} : \frac{5}{12}; \quad d. \frac{5}{9} \cdot 2\frac{1}{4} : 20;$$

14. Evaluate:

$$a. 14 : 42; \quad b. 2 : 3 : 5; \quad c. 2 : 8 \cdot 3; \quad d. 100 \cdot 6 : 40; \quad e. 5 : 15 \cdot 3.$$

15. Melon weighs 7 pounds, and the watermelon is $1\frac{1}{5}$ times heavier. How many pounds is watermelon is heavier than the melon?

16. $4\frac{1}{2}$ kg. of candies were packed into $\frac{1}{2}$ kg packages. How many packages were the candies packed into?

17. Find the unknown:

$$a. \frac{1}{3} \cdot x = \frac{1}{6}; \quad b. \frac{2}{5} \cdot x = 1\frac{1}{5}; \quad c. \frac{2}{3} \cdot x = 1$$

$$d. x \cdot 6 = 1\frac{1}{5}; \quad e. x \cdot 6 = 4; \quad f. 3 \cdot x = \frac{1}{3}$$

18. There are 32 kg of apples in two baskets. There are 4 times as many apples in one basket as in the other. How many kilograms of apples are there in each basket?

19. While doing his homework, John noted the time spent on preparing each lesson: working with a map, solving a problem, and memorizing a poem. Using the data obtained, he formulated two problems. Solve them and try to create problems yourself using your own data:

- a. The student spent $\frac{1}{4}$ hour on geography and math problem, with the work on the map taking $\frac{1}{20}$ hour less than solving the problem. How much time was spent on each task?
- b. The student spent $\frac{2}{5}$ hour on working with the map and memorizing the poem, with three times as much time spent on memorizing the poem as on working with the map. How much time did each task take?
20. The father is 40 years old. The son's age is $\frac{3}{8}$ of the father's age. How old is the son?

$$\begin{array}{r} \square\square\square \\ 8 \overline{) \square\square\square\square} \\ \underline{-3\square} \end{array}$$

21. The son is 10 years old. His age is $\frac{2}{7}$ of the father's age. How old is the father?

$$\begin{array}{r} 2\square \\ -\square\square \\ \hline 0 \end{array}$$

22. Fill in the empty spaces:

23. Can you say which of the following statements are true and which are false?
- If a natural number is divisible by 4 and 3, it's divisible by 12
 - If a natural number is divisible by 12, it's divisible by 3 and 4.
 - If a natural number is divisible by 9, it's divisible by 3.
 - If a natural number is divisible by 3, it's divisible by 9.

24. 25 identical thick books or 45 identical thin books can fit on a bookshelf. Will there be enough space on a bookshelf for 20 thick and 9 thin books?

25. Compare fractions:

$$a. \frac{1}{2} + \frac{1}{5} \quad \text{and} \quad \frac{1}{3} + \frac{1}{4}; \quad b. \frac{1}{2} - \frac{1}{3} \quad \text{and} \quad \frac{1}{4} - \frac{1}{5};$$

26. One natural number is 4 greater than the other. Find these numbers if their product is 96.

27. There are tents and cabins at the campground, with a total of 25 units. There are 4 people living in each cabin, and 2 people in each tent. How many tents and how many cabins are there at the campground if there are a total of 70 people staying there?

28. Find LCM and GCF of the numbers:

$$a. 72 \text{ and } 12; \quad b. 16, 28, \text{ and } 32; \quad c. 8, 9 \text{ and } 25.$$

29. Poles are placed along the road, starting from point A, every 45 meters. It was decided to replace these poles with others, placing them 60 meters apart. Find the distance from point A to the nearest pole that will stand in the place of the old one, except for the pole at point A.

30. For a field trip, several buses were allocated to the schools, each with the same number of seats. 424 students from the elementary school went to the forest, and 477 students from the middle school went to the lake. All seats in the buses were occupied, and no one was left without a seat. How many buses were allocated, and how many passengers were on each bus?