Math 4a, classwork 15.

Variables.

Fill the form:

The color I like is ___________________________.

I have a ___________________________ as my pet.

*if you don't have any pets, leave blank.

The subject I like the most in school is ______________________ .

In this form, blank spaces are provided to fill in the information when it becomes available; these blank spaces are used in the same way as variables in mathematics.

When we need to write the mathematical expression, but we don't know exact numbers to be used, we use variables. It can be any symbol, but it's very convenient to use letters. For example, if the number of the books on one shelf is n and the number of the books on the other shelf is m, the total number of books on both shelves is $n + m$. We can perform all the usual arithmetic operations with the variables, but the exact answer can be reached only when the values are assigned to the variables.

Write the expression for the following problems:

3 packages of cookies cost a dollars. How many dollars do 5 of the same packages cost?

If 3 packages of cookies cost a dollars, the price of one pack is

$$1pack = \frac{a}{3} = a:3 = \frac{1}{3}a$$

Five such packs will cost

$$5 \cdot a:3 = \frac{5a}{3} = \frac{5}{3}a$$

5 bottles of juice cost b dollars. How many bottles can one buy with c dollars?

Again, if 5 bottles cost b dollars, the price of one bottle is

$$\frac{b}{5} = \frac{1}{5}b \text{ dollars}$$

If I have only c dollars, I can buy the number of bottles equal to my total money divided by the price of one bottle:

$$c : \frac{b}{5} = c \cdot \frac{5}{b} = \frac{5c}{b}$$

If I have only \$30 and 5 bottles cost 10 dollars I can buy:

$$30 : \frac{10}{5} = 30 \cdot \frac{5}{10} = 30 \cdot \frac{1}{2} = 15 \text{ bottles}$$

Mia read m books during the summer, Lia read p books, and Peter read $\frac{3}{4}$ of the number of books that Mia read. How many books did they read altogether during the summer vacation? Solve the problem if $m = 12$ and $p = 15$.

Number of books that Peter read is

$$\frac{3}{4}m$$

Altogether, they read

$$m + p + \frac{3}{4}m$$

We can simplify this expression, because like terms can be combined.

$$m + p + \frac{3}{4}m = \frac{7}{4}m + p$$

To find the exact answer to the problem, values should be assigned the variables m and p .

$$\frac{7}{4} \cdot 12 + 15 = 7 \cdot 3 + 15 = 21 + 15 = 36$$

Equation.

Equation is an equality with one or more variables. For example:

$1 + x = 15;$ $\quad x + y + z = 100$ $\quad z \cdot y = 1$ $\quad |x| = 10$ $\quad |x| = -10$

To solve an equation means to find all possible values of the variable(s) that the equation will become a true equality.

In the first equation above, if the value of x is 14, the right side of the equality is equal to the left side:

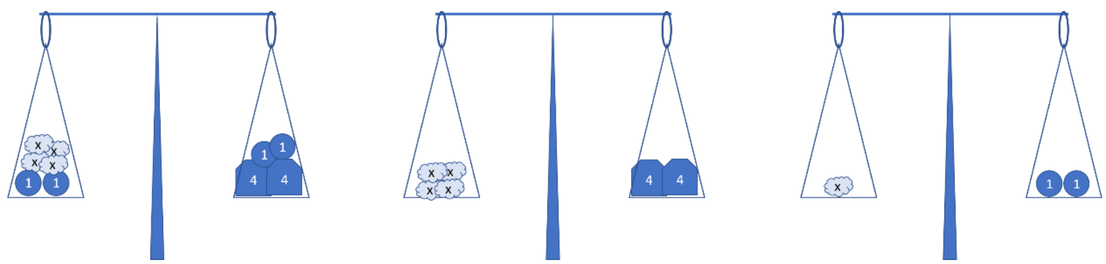
$1 + 14 = 15$. The equation $|x| = -10$ has no solution for x, because there is no number whose absolute value is negative. There are an infinite number of triples for x, y, and z whose sum equals to100. One possibility, for example, is $x = 25, y = 25, z = 50$

Another examples of equations:

$$5y = -100$$
$$4 + 3x = 25$$
$$34 = 2x + 4$$
$$2 + 4x = 10$$

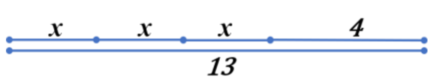
What can we do to solve an equation? For example, let's solve the equation $2 + 4x = 10$. We can add (or subtract) the same quantity to (or from) both sides of the equation while maintaining the balance. Similarly, we can multiply or divide both sides by the same value, keeping the balance intact.

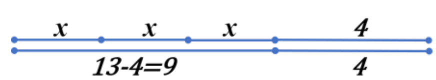
$2 + 4x = 10$ $\quad$ $2 - 2 + 4x = 10 - 2$ $\quad$ $(4:4)x = 8:4$



The process of the solving equation can be visualized in a different way. Let solve another equation:

$3x + 4 = 13$
$3x = 13 - 4 = 9$
$x = 9:3 = 3$





Substitution.

Let’s take a look at a very simple equation.

$$|x| = 10$$

The solution to this equation is a number, the absolute value of which is 10. There are two such numbers, 10 and (-10). Thus, this equation has two roots. (The word “root” can be used as a synonym for solution). Another equation:

$$|x| + 5 = 10$$

To make the equation a little simpler, we can substitute $|x|$ with m ($|x| = m$) and solve for m.

$$m + 5 = 10$$

$$m + 5 - 5 = 10 - 5$$

$$m = 5.$$

But the initial variable is x, not m. $|x| = m$, or, as we know, $|x| = 5$. There are two roots, 5 and (-5).

Equations are very useful to solve word problems. In each word problem there is an unknown quantity, and known parameters out of which the equation can be created. For example, let's take a look on the following problem:

There are 27 pencils in two boxes altogether. There are 5 more pencils In one box then in the other. How many pencils are there in each box?

There are two unknown quantities in this problem, the number of pencils in the first box and the number of pencils in the second box. But these two quantities are not independent, one is 5 less than the other. If the number of pencils in one box is denoted as x, number of pencils in the second box will be $x + 5$. And we also know that the total number is 27.

$$x + x + 5 = 27$$

$$2x = 27 - 5 = 22$$

$$x = 22: 2 = 11$$

Answer: there are 11 pencils in one box, and 16 in the other.

There are candies in a box. If each kid will take 4 candies, 19 candies will be left in the box. If each kid will take 5 candies, there will be lacking 2 candies. How many candies are there in the box?

In this problem there are also two unknown quantities, the number of kids, and number of candies in the box. If the number of kids is denoted as x, the number of candies can be calculated in to ways:

First, $5 \cdot x - 2 =$ number of candies in the box

Second, $4 \cdot x + 19 =$ number of candies in the box, so

$$5 \cdot x - 2 = 4 \cdot x + 19$$

$$5x - 4x - 2 + 2 = 4x - 4x + 19 + 2$$

$$5x - 4x = 19 + 2$$

$$x = 21$$

The number of kids is 21. The number of candies can be calculated from either expression:

$5 \cdot 21 - 2 = 4 \cdot 21 + 19 = 103$

Answer: there are 103 candies in the box.

There were 624 books in two boxes altogether. When $\frac{1}{3}$ of the books from one box and $\frac{3}{7}$ of the books from another box were sold to the customers, the number of books in each box became equal. How many books there were in each box at the beginning?

In this problem there are two unknown variables, number of books in each box. Let's denote the number of books in the first box as x, and the number of books in the second box as y. Together $x + y = 624$. But we know also that

$$\frac{2}{3}x = \frac{4}{7}y$$

$$x = \frac{4}{7}y \cdot \frac{3}{2} = \frac{4 \cdot 3}{7 \cdot 2}y = \frac{6}{7}y$$

We can now substitute x in the equation $x + y = 624$ with $\frac{6}{7}y$.

$$\frac{6}{7}y + y = 624$$

$$\frac{13}{7}y = 624$$

$$y = 624 \cdot \frac{7}{13} = 48 \cdot 7 = 336$$

$$x = \frac{6}{7} \cdot 336 = 288$$

Answer: 288 books, and 336 books.

On the lawn grew 35 yellow and white dandelions. After eight whites flew away, and two yellows turned white, there were twice as many yellow dandelions as white ones. How many whites and how many yellow dandelions grew on the lawn at the beginning?

Again, there are two unknown amounts in the problem: number of yellow and number of white dandelions at the beginning, the sum of these two numbers is 35. We can use y and w as variable names for convenience.

$$y + w = 35$$

Which gives us the following relationship:

$$w = 35 - y$$

Also, we know that

$$2 \cdot (w - 8 + 2) = y - 2$$

$$2(w - 6) = y - 2$$

(eight whites are gone and two yellows are now white, and number of yellows now twice as big as number of whites). Using the substitution $w = 35 - y$, the last equation can be rewritten as

$$2(35 - y - 6) = y - 2$$

$$2(29 - y) = y - 2$$

$$58 - 2y = y - 2$$

$$58 + 2 = y + 2y$$

$$3y = 60$$

$$y = 20, \quad w = 35 - 20 = 15$$

Answer: at the beginning there were 15 white and 20 yellow dandelions.

Do you have any idea how to solve this problem without writing un equations?

Mary bought 5 apples and 2 pears for $7.90. Eva bought 8 apples and 6 pears for $16.14. Veronica bought 3 apples and 3 pears. How much change did she get back from $10.00?

Let's denote the price of an apple as a and the price of a pear as p. The price of 5 apples can be written as $5a$ and the price of 2 pears is $2p$.

$$5a + 2p = 7.9 \qquad (1)$$

Also

$$8a + 6p = 16.14 \qquad (2)$$

Can the price of one pear be expressed through the price of an apple from the first equation? Two pears cost exactly \$4.60 minus the price of 5 apples. One pear costs half of it:

$$2p = 7.90 - 5a; \quad p = \frac{1}{2} \cdot (7.90 - 5a) = 3.95 - \frac{5}{2}a$$

To calculate the price of six pears we even don't need to write the expression for the price of one pear, six pears is three times more expensive than two pears.

$$6p = 2p \cdot 3 = (7.70 - 5a) \cdot 3 = 23.7 - 15a$$

But from the second equation we know that

$$8a + 6p = 16.14;$$

$$8a + 23.7 - 15a = 16.14$$

$$8a - 15a = 16.14 - 23.7$$

$$15a - 8a = 23.7 - 16.14; \qquad 7a = 7.56$$

$$a = 7.56 : 7 = 1.08$$

After the price of an apple is known, all other variables can be found easily and problem will be solved.

Exercises:

1. An apple costs x dollars and a pear cost y dollars. Explain the expressions below:

$$x + y, \qquad x - y, \qquad 3x, \qquad 8y, \qquad 3x + 8y, \qquad y : x, \qquad 120 : y$$

2. Write the following as mathematical expression. If this expression is an equation, solve it.

   a. Sum of the number *x* and 15 equals to 20.

   b. Product of *y* and10.

   c. Difference between three times *z* and 4 is equal to12.

   d. Half of the number *b* is equal to 1.5

   e. Product of the numbers of 5 and *x* is less than 12.

3. 10 identical notebooks cost *x* dollars. Textbook costs 15 dollars more than notebook.

   a. What is the price of one notebook?

   b. What is the price of the textbook?

   c. What is the price of n notebooks?

   d. What is the price of n notebooks and m textbooks?

4. Peter had $\boldsymbol{n}$ dollars to buy lunch. He spent $\boldsymbol{k}$ dollars on a salad and three times as much on a burger. How much money does he have left? Write the expression to solve the problem.

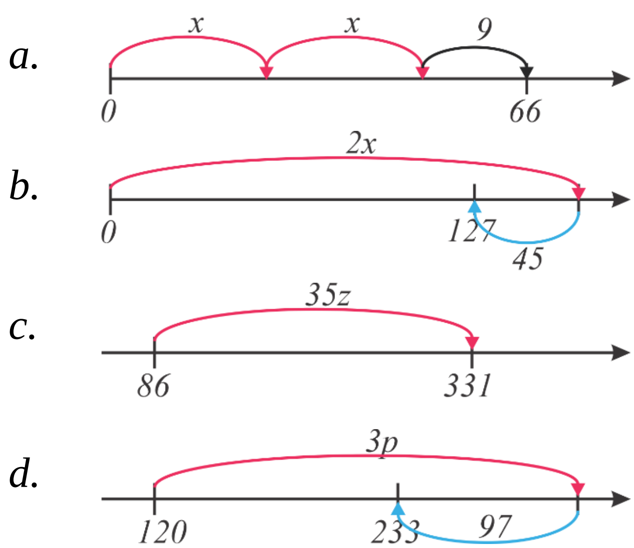
5. A farmer got $\boldsymbol{d}$ bags of potatoes from the first field. From the second field, he got $\boldsymbol{x}$ bags more than from the first, and from the third field, he got $\boldsymbol{b}$ bags less than from the first field. How many bags of potatoes did he get altogether? **Write an expression first**, then solve the problem using the following values for the variables: $\boldsymbol{d} = 685,\ \ \boldsymbol{x} = 4,\ \ \boldsymbol{b} = 56$

6. Come up with a problem, the solution to which can be described by the following expression:

   $a.\ \ a + b : 3;$ $\qquad$ $b.\ \ (a + b) : 3;$ $\qquad$ $c.\ \ a + b \cdot 3;$ $\qquad$ $d.\ \ (a + b) \cdot 3$

7. The sum of three consecutive odd numbers is 135. What is the smallest of the three numbers?

8. Based on the drawing below, write equations and solve them:



9. Mary bought 5 apples and 2 pears for $7.90. Eva bought 8 apples and 6 pears for $16.14. Veronica bought 3 apples and 3 pears. How much change did she get back from $10.00?

10. There were 624 books in two boxes altogether. When $\frac{1}{3}$ of the books from one box and $\frac{3}{7}$ of the books from another box were sold to the customers, the number of books in each box became equal. How many books there were in each box at the beginning?

11. Solve the following equations:

    $a.\ \ 2x + 3 = 11;$ $\qquad$ $b.\ \ \dfrac{1}{2}x - 5 = 12;$ $\qquad$ $c.\ \ 14 + x = 4 + 6x$