

**MATH 10**  
**ASSIGNMENT 12: MATH BATTLE!!**  
 DEC 15, 2024

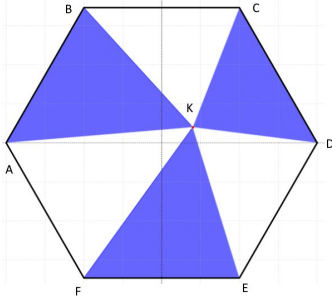
1. Let  $a_n$  be the sequence defined by the rules

$$a_1 = a_2 = a_3 = 1$$

$$a_n = a_{n-1} + a_{n-2} + a_{n-3} \quad \text{for } n \geq 4$$

Find  $a_{2024} \pmod{12}$ .

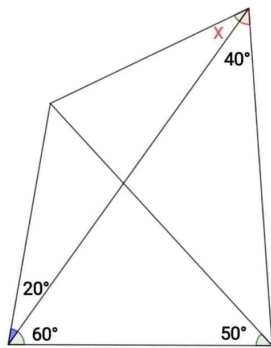
2. Shurik has chosen a point inside a regular hexagon and connected it to each vertex. He colored the resulting six triangles white and blue as shown in the figure. Prove that the total area of blue triangles is equal to the total area of white triangles.



3. A spiderweb is a square with  $100 \times 100$  nodes (thus with  $99 \times 99$  cells). 100 flies are caught in the web, stuck at 100 different nodes. A spider which was originally in a corner of the web crawls from a node to an adjacent node counting steps and eating flies on its way. Can the spider eat all the flies in no more than 2100 steps?
4. In 3d space, we have marked points  $A_1, A_2, \dots, A_n$  and a plane  $P$  so that the plane intersects each of the segments  $A_1A_2, \dots, A_{n-1}A_n, A_nA_1$ : it intersects  $A_1A_2$  at point  $B_1, \dots, A_nA_1$  at point  $A_1$ . Prove that then

$$\frac{A_1B_1}{A_2B_1} \frac{A_2B_2}{A_3B_2} \dots \frac{A_nB_n}{A_1B_n} = 1$$

5. Find angle  $x$  in the picture



**Find X**