ASSIGNMENT 1: MISCELLANEOUS PROBLEMS
SEPTEMBER 15, 2024

1. During daytime, a snail climbs 10cm up a post. During the night, it slides down 9cm. How long will it take the snail to reach the top of the pole if the height of the pole is 1m?

2. A rectangular bar of chocolate consists of $m \times n$ squares. You want to break it into mn individual squares. At each step, you may pick up one piece you have and break it along any of the vertical or horizontal lines separating the squares.
   How many breaks do you need? What is the fastest way to do it?

3. Alexander has a bucket of wine and bucket of water. He draws 1 quart of wine from the wine bucket and pours it into the bucket with water, carefully stirs it and then draws 1 quart of the mix and pours it into the wine bucket.
   What is larger: amount of wine in the water bucket or amount of water in the wine bucket?

4. A rook is placed in the lower left corner of the usual 8×8 chessboard. Can you move it to the upper right corner so that on the way, it visits every square of the chessboard exactly once? (Going over some square also counts as visiting.)
   What about a knight?

5. If we draw 4 planes through the origin in 3d space, into how many regions will they divide the space? what about n planes?
   The planes are supposed to be "in general position": no three planes have common points other than the origin.

6. You have two identical glass balls. Your goal is to find the maximal height from which these balls can be dropped without breaking. To do that, you can drop the balls (one at a a time) from any floor of a 100-story building.
   How many attempts will you need?

7. Solve the equation
$$2024 - 2(2024 - 2(2024 - 2(2024 - 2x))) = x$$

8. Prove that the number $30^{239} + 239^{30}$ can not be prime. [Hint: 31 is prime]

9. Kathryn has drawn on the blackboard a 4×6 grid. Nestor wants to erase some segments so that one can still reach any of the intersection points of the original grid, by starting at the corner and following the chalk lines. What is the maximal number of segments Nestor can erase? (Or, if it is easier: what is the smallest number of segments he can leave?)
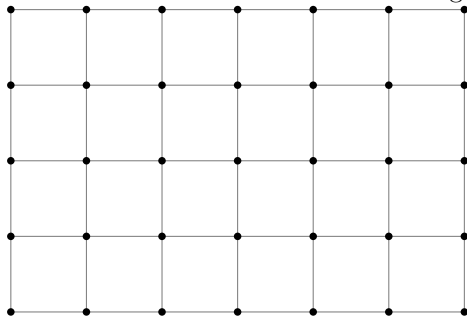
10. You are given a straightedge (which allows to draw lines through two given points, but has no marks, so you can not measure distances) and a "right angle" — a tool that allows constructing the perpendicular to the given lile through a given point **on this line**.
   Using these tools, can you construct a perpendicular to the given line through a point **not on this line**?