Homework 13.

Time dilation

The result of Michelson-Morley experiment indicate that the speed of light is the law of nature and is the same in all inertial reference frames. The idea of universality of the speed of light was proposed by a famous French mathematician Henri Poincaré and Albert Einstein. Let us try to see what happens if we assume that the speed of light is absolute and does not depend on how fast the source of the light is moving with respect to you.
Let us imagine a train carriage moving to the right with a velocity V (see Figure 1).

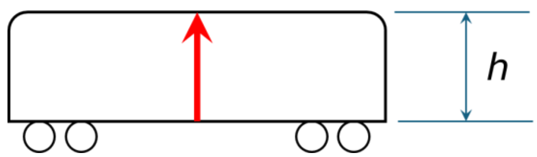

Figure 1.

At a certain moment, a flashlight at the floor of the carriage is turned on an emits the light vertically up, towards the ceiling. For a person in the carriage the light is traveling vertically up and reaches the ceiling in a time

$$c = \frac{h}{\Delta t} \qquad (1),$$

Where h is the height of the carriage ceiling and c is the speed of light in the carriage. From the prospective of the person standing on the ground, the light travels along a slanted line since the carriage and the flashlight are moving with respect to the ground (Figure 2).

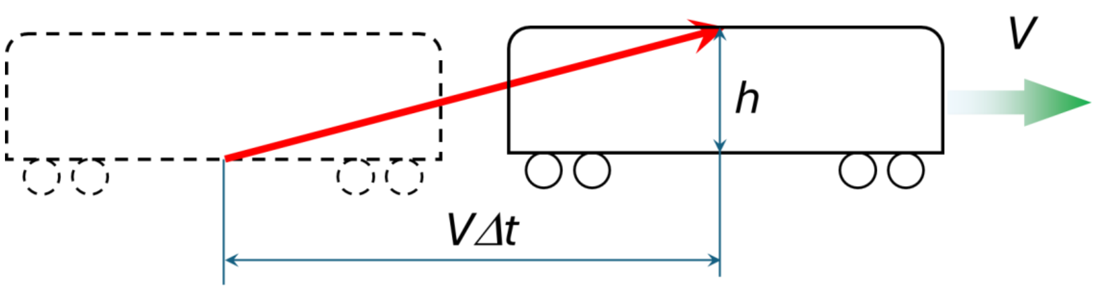

Figure 2.

While he light is traveling from the carriage floor to the ceiling, the carriage shifts to the right by $\Delta d = V \cdot \Delta t$, so the distance, traveled by the light can be found using Pythagorean theorem:

$$\Delta l = \sqrt{h^2 + (V \cdot \Delta t)^2} \qquad (2).$$

If we calculate the speed of light in the reference frame of the ground, we have:

$$c_{ground} = \frac{\Delta l}{\Delta t} = \frac{\sqrt{h^2+(V\cdot\Delta t)^2}}{\Delta t} = \sqrt{c^2 + V^2} \qquad (3).$$

According to the expression (3), the calculated speed of light in the stationary reference frame should be higher, than that in the reference frame of the carriage. But this is in contradiction with Michelson -Morley experiment! According to this experiment, c_{ground} should be equal to c. But how we can do that? Expressions (1) and (3) apparently are not equal to each other. To make them equal we have to change something. A crazy suggestion was that the time flows differently for an observer at the ground and an observer who is moving with respect to the ground., So Δt in the expressions (1) and (3) are different! Let us denote the time interval in a moving reference frame as $\Delta t'$ and make the speed of light in the moving carriage and in the stationary reference frame to be equal to each other:

$$c = \frac{h}{\Delta t'} = \frac{\sqrt{h^2+(V\cdot\Delta t)^2}}{\Delta t} = \sqrt{\left(\frac{h}{\Delta t}\right)^2 + V^2} \qquad (4).$$

Taking into account that $h = c \cdot \Delta t'$, we have:

$$c = \sqrt{\left(\frac{c\cdot\Delta t'}{\Delta t}\right)^2 + V^2} \qquad (5).$$

Expressing $\Delta t'$ we obtain:

$$\Delta t' = \Delta t\sqrt{1 - \frac{V^2}{c^2}} \qquad (6).$$

According to (6), time in a moving reference frame flows slower! This effect is known as the *time dilation*. There are a lot of experimental justifications of the time dilation. One is the Rossi-Hall experiment. In 1940, at Echo lake (elevation 3420m) in Denver, CO, B. Rosi and D. B. Hall were measuring relativistic decay of muons. Muons are massive negatively charged particles with a relatively short lifetime of 2.2 ns. Created in the upper atmosphere by cosmic rays, they are moving at a high speed of 0.995c. But even at this high speed the distance they can pass during their lifetime is about 650m, so no muons are expected to reach the earth. However, Rossi and Hall were able to register a lot of muons near the earth surface. This proves the time dilation effect, since from the perspective of an observer at rest, the time of a rapidly moving muon flows slower.

Problem:

Calculate the muon lifetime as it is “seen’ by the observer at the ground. What is the distance traveled by the muon from this observr’s perspective?