Homework 7.

Torque

During last class we discussed torque. For rotational dynamics torque plays same role as force plays for the linear dynamics. If a nonzero torque applied to an object, the object experience angular acceleration β. Simply speaking, to turn an object we have to apply torque to it.

Imagine that you are trying to rotate a rigid body (Fig. 1) with respect to the axis passing through the point A ("pivot point") and perpendicular to the page plane.
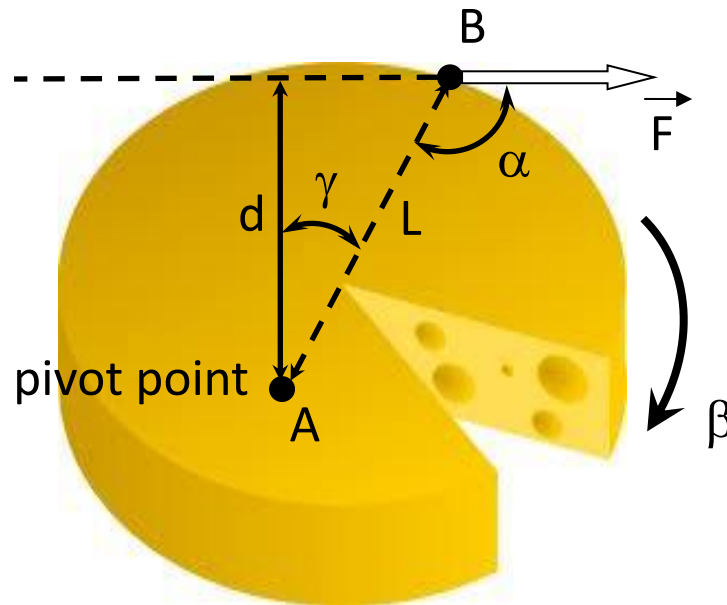

Figure 1.

$$\left|\vec{T}\right| = d \cdot \left|\vec{F}\right| \qquad (1)$$

Torque can be calculated as a product of the force magnitude and the distance d between the pivot point and the straight line which passes through the point to which the force is applied (point B in Fig.1) and parallel to the force direction. Torque can also be calculated through the distance L between pivot point and the point of the force application. As we can see from the Figure 1, $d = L \cdot \cos\gamma = L \cdot \sin\alpha$. So

$$\left|\vec{T}\right| = \left|\vec{F}\right| \cdot L \cdot \sin\alpha \qquad (2),$$

where α is the angle between the force direction and the line connecting pivot point and point to which the force is applied. Torque is a vector – in a sense that it has both magnitude and direction. Let us consider the sign of the torque as the indication where it "wants" to turn the object – clockwise or counterclockwise. If we know total net torque T_{tot} which applied to the body, we can calculate the angular acceleration β:

$$\overrightarrow{T_{tot}} = I \cdot \vec{\beta} \qquad (3),$$

where I is the moment of inertia. *Please note that both torque and the moment of inertia in the formula (3) should be calculated with respect to same axis.* If you use a different axis both torque and moment of inertia can be different. Compare formula (3) with the Newton's second law:

$$\overrightarrow{F_{tot}} = m \cdot \vec{a} \qquad (4),$$

Where m is mass, a – acceleration, F_{tot} – total net force applied to the body.

Problems:

1. A cylindrical coil of a radius R with thread can rotate without friction around the coil's "main' axes. You are pulling the thread with a constant force F. The mass of the coil is m (the thread is very light). What is angular velocity of the coil after you pulled out a thread's tail of the length l? (See picture below).
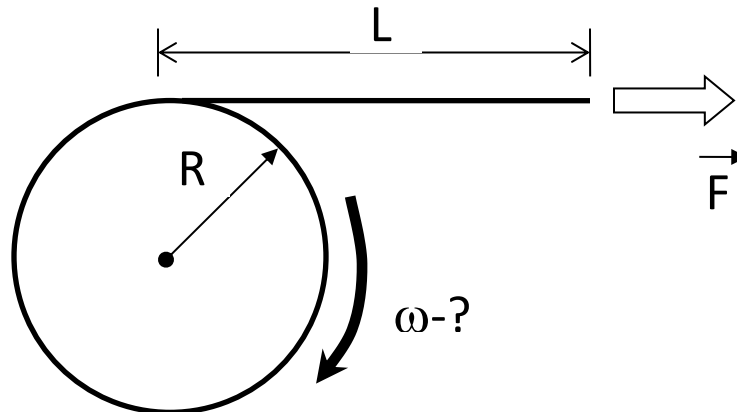
2. Find final kinetic energy of the coil.