ACCELERATION

OCTOBER 1, 2023

Theory recap

In everyday life we use the word acceleration to describe increase of the speed of a moving object. Acceleration in physics has different meaning. It is change in *velocity* per unit time. Any time the speed and/or the direction of motion of an object changes we deal with accelerated motion. There are many examples of accelerated motion: a bike starting moving from rest, a car braking, any object falling.

Acceleration as a physical quantity is defined as the rate of change of velocity. It means that if during a time interval t the velocity of an object changed by Δv (here Δ is the Greek letter Delta, which is often used in physics to denote the change in some quantity), the acceleration a of the object is found as follows:

$$a = \frac{\Delta v}{t}.$$

For instance, we know that any object (if we could neglect the air drag) falls down with acceleration of approximately 10 $\mathrm{m/s^2}$ (9.8 $\mathrm{m/s^2}$, to be exact). This means that if we drop an object without any initial speed, after 1 second of free fall it will gain speed 9.8 m/s, after 2 seconds it will be $9.8 \times 2 = 19.6$ m/s, after 3 seconds $9.8 \times 3 = 29.4$ m/s etc. (until it hits the ground, of course).

Acceleration is a vector - it has both magnitude and direction. Because of this a slightly better way of writing our formula for acceleration would be

$$\vec{a} = \frac{\Delta \vec{v}}{t}$$

Here we emphasize with arrows that both acceleration and velocity are vectors while time is a scalar. We can play a bit with this formula and rearrange it. Suppose that our question is to find velocity $\vec{v}$ of an object moving with acceleration $\vec{a}$ if we know its initial velocity $\vec{v_0}$ and the time t it has been accelerating. The change in velocity is the difference between the final velocity and the initial velocity:

$$\Delta \vec{v} = \vec{v} - \vec{v_0}.$$

We can make a chain of simple operations to our formula of acceleration:

$$\vec{a} = \frac{\vec{v} - \vec{v_0}}{t} \Longrightarrow \vec{a}t = \vec{v} - \vec{v_0} \Longrightarrow \vec{a}t + \vec{v_0} = \vec{v}.$$

We have obtained an important formula:

$$\vec{v} = \vec{v_0} + \vec{a} \cdot t \tag{1}$$

Velocity after the time t = Initial velocity plus **acceleration** multiplied by the **time**

For example, if you just let a pebble go down, the initial velocity is zero. But you can throw the pebble down. In this case the pebble starts accelerating from nonzero velocity.

Just to remind, arrows on top of some characters in formula (1) mean that the corresponding physical parameters are vectors. Because of this for the case of rectilinear motion (just to remind - this is the motion along a straight line) there are two major cases:

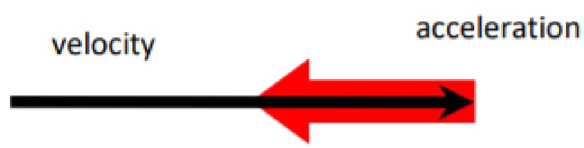
1. Acceleration is directed along the initial velocity (like when using a gas pedal in a car).

   In this case velocity and acceleration have the same sign and speed of the object is *increasing* with time. The acceleration magnitude gives us the rate of the speed increase. For example acceleration of 5 meters per second per second (this is not a typo!) means that the speed increases by 5 m/s every second. It is denoted as 5 m/s^2 (five meter per second squared). All vectors in formula (1) point in the positive direction, so we take them with a plus sign:

$$v = v_0 + at.$$

   This formula describes speed growing with time.

2. Acceleration is directed oppositely to the initial velocity (like using a brake pedal).



   In this case velocity and acceleration have the opposite signs and speed of the object is *decreasing* with time. The acceleration magnitude gives us the rate of the speed decrease. For example, acceleration of -5 meters per second per second means that the speed decreases by 5 m/s every second. Let us pick the positive direction along the initial velocity. Now in formula (1) v_0 and v are taken with a plus sign while a is taken with a minus sign because is points opposite to the positive direction:

$$v = v_0 - at.$$

   This formula describes speed decreasing with time.

Homework

When solving these problems you can round the acceleration due to gravity to 10 m/s^2.

1. Imagine that you dropped a penny from the Empire State Building (please, never do it in real life!). Calculate the speed of the coin in 5 seconds.
2. a) Tesla's Model S Performance is the fastest-accelerating production car. It reaches 60 mph in just 2.4 seconds. Find its acceleration and compare it to the free fall acceleration on Earth, g (note that you will need some unit conversion to carry out the comparison!).

b) Now assume that a car traveling at a speed 60 mph starts braking with acceleration -4 m/s^2. How long would it take to stop?

3. A ball with zero initial velocity falls down from a height of 5m and hits the ground in 1 second. Find average velocity of the ball and compare it to the velocities of the ball in the beginning and in the end of the motion.