November 12, 2023

Geometry.

Recap: The Inscribed Angle Theorem.

Theorem. An angle α inscribed in a circle is half of the central angle $\beta = 2\alpha$ that subtends the same arc on the circle (Fig.1), or complete half of it to 180.
Corollary. The angle does not change as its apex is moved to different positions on the circle.

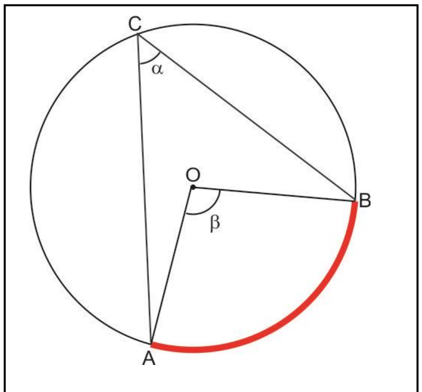


Fig. 1

Proof. First, let us deal with the simple case when one of the rays of angle ACB' passes through the center of the circle (Fig. 2). $\angle AOB'(\beta)$ is a central angle that subtends the same arc as $\angle ACB'$ (α). Triangle AOC is an isosceles triangle because $|OA| = |OC|$, so angle $\angle OAC$ and angle $\angle OCA$ are equal and angle $\angle AOC = 180 - 2\alpha$, but it is also equal $180 - \beta$ as a supplement angel to angle β.

$$\angle AOC = 180 - 2\alpha = 180 - \beta \Rightarrow \beta = 2\alpha.$$

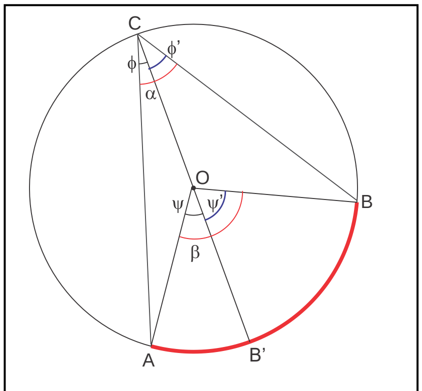


Fig. 2

In the case when center of the circle placed inside of angle ACB we can divide the angle ACB with a ray CB' passing through the center of the circle (Fig. 3). Now we have two inscribed angles: angle ACB' and angle $B'CB$, each of them has one side which passes through the center of the circle and can use previous part to proof that $\beta = 2\alpha$.

$$\alpha = \phi + \phi',$$

$$\beta = \psi + \psi' = 2\phi + 2\phi' = 2(\phi + \phi') = 2\alpha.$$

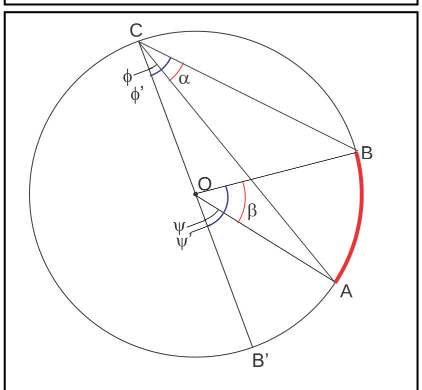


Fig. 3

When center of the circle is outside of inscribed angle, we can draw a ray from a vertex of our angle through the center the circle (Fig. 4). Then the angle $\angle ACB(\alpha) = \angle B'CB(\phi') - \angle B'CA(\phi)$ and we again can use the first part.

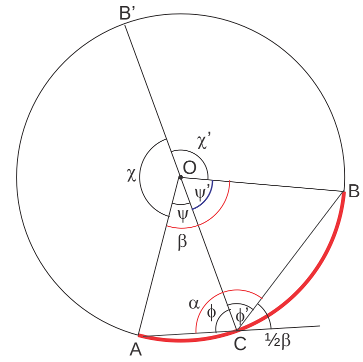


Fig. 4

$$\beta = \psi' - \psi = 2\phi' - 2\phi = 2(\phi' - \phi) = 2\alpha.$$

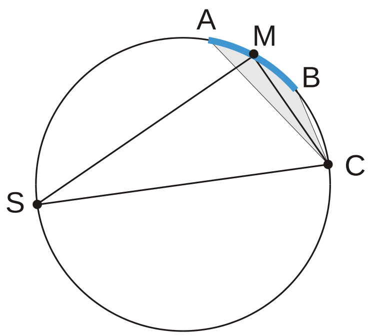
Only the case of obtuse angle is left. In this case the ray CB' passes through the center of the circle and divides angle ∠ACB into two angles ϕ and ϕ' They are not now half of the angles ψ and ψ', but half of their supplement angles χ and χ' therefore,

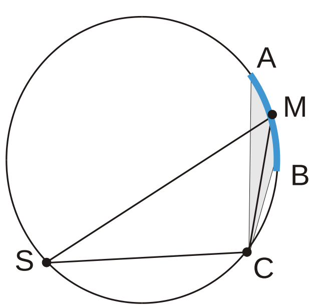
$$\alpha = \frac{1}{2}\chi + \frac{1}{2}\chi' = \frac{1}{2}(\chi + \chi') = \frac{1}{2}(180 - \psi + 180 - \psi') = 180 - \frac{1}{2}(\psi + \psi') = 180 - \frac{1}{2}\beta.$$

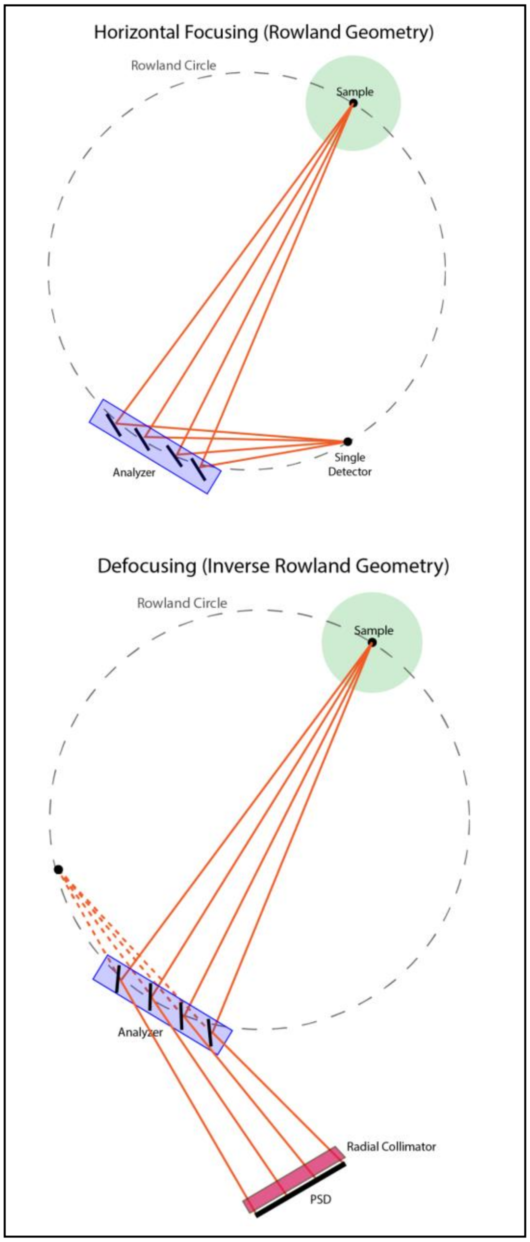
The Rowland circle.

In scientific diffraction instruments, it is often desirable to have a diffraction mirror (grating) shaped in a way such that a beam of light, or particles, emanating from a point source is focused to a point being reflected monochromatically by that diffraction mirror. That is, the angle between the incident and the reflected (diffracted) beams is the same for any point on the mirror. Such mirror is a segment of the so-called Rowland circle.

Consider the schematics of the Rowland-circle focusing setups shown below and try to imagine how these setups function for monochromatic focusing and how the devices shown in the pictures below fit in these schematic setups.






Horizontal Focusing (Rowland Geometry)
Rowland Circle
Sample
Analyzer
Single Detector
Defocusing (Inverse Rowland Geometry)
Rowland Circle
Sample
Analyzer
Radial Collimator
PSD