ASSIGNMENT 15: TOURNAMNET OF TOWNS

APRIL 6, 2024

Here are some problems from an international math contest *Tournament of Towns*. You can find more info about it here: https://www.turgor.ru/en/

Note: during the contest, you are given 5 hours to solve the problems.

1. Shurik is cutting his birthday cake.

He can take the whole cake or one of the pieces obtained from previous operations and cut it either into two **equal** parts, or into more than 2 parts, which will all be **different**. For example, a piece of weight 10 can be cut into 5 and 5, or into 1,3, and 6 (or in many other ways).

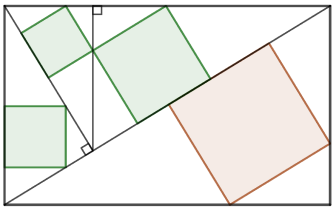
Is it possible that after some number of such operations, the original cake is cut into 17 equal parts?

2. An 8×8 chessboard is colored using several colors (each square is single color) so that whenever two squares are **diagonally** adjacent (such as a1 and b2) or related by a knight's move (2 steps in one direction and one in perpendicular direction), these squares are of different colors.

What is the smallest number of colors that could have been used?

3. A rectangle is cut into four right triangles as shown in the figure below. Into each triangle we inscribe a square so that the one side of the square lies on the hypotenuse.

What is greater, the area of the largest square (red in the picture) or the sum of areas of the three remaining ones?



4. Two players are palying the following game. They have a pile of 100 stones. The first player takes 1 stone, then the second palyer can take 1 or 2 stones, then the 1st player can take 1, 2, or 3 stones, etc.

Whoever gets the last stone, wins.

Is there a winning strategy for one of the players?