PRESSURE

MARCH 23, 2023

Pressure. From daily life we know that in some situations not just the force matters but also to which area this force is applied. Previously we did not discuss where exactly is a force applied, but in fact it never could be in just a point - there should be some area. When a person stands on the floor, the normal force which the floor exerts on the person is applied throughout the whole area of their feet. It is thus important to consider pressure (denoted by p), which is defined as a ratio of force F to the area A to which this force is applied:

$$p = \frac{F}{A}$$

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Units of pressure are $\mathrm{N/m^2}$ and have a special name: Pascals, or Pa. For example, let's calculate the pressure a person exerts on the floor when standing on two feet. Let us say for simplicity that each of the feet is a rectangle with sides 30 cm $\times$ 10 cm and mass of the person is 60 kg. Then pressure is

$$p = \frac{F}{A} = \frac{mg}{A} = \frac{60 \cdot 10 \text{ N}}{2 \cdot 10 \cdot 30 \text{ cm}^2} = \frac{600}{0.06} \frac{\text{N}}{\text{m}^2} = 10000 \text{ Pa} = 10 \text{ kPa}$$

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Normally we do not care too much about this pressure. However if we try to walk on snow it becomes important: if the pressure is too big, our feet will fall through the snow. In order to reduce pressure and prevent falling through the snow one could make the area of contact bigger by wearing snowshoes or skis.

Pressure in fluids. A fluid is a term which means either gas or liquid. We want a common name for them, because they share a property of taking the shape of the container which means that they flow (hence the name).

First, consider a cylindrical vessel with liquid. Let the cross section area of the vessel be A, height of the liquid level above the bottom be h, and density (mass per unit volume) of the liquid be ρ. We are interested in finding the pressure of the liquid at the bottom of the vessel. In order to do it let us consider all forces acting on the liquid. There are two forces: gravity force mg and normal force from the bottom of the container N. Mass of the liquid is

$$m = \rho V = \rho A h$$

where V is volume of liquid, equal to product of cross section area and height. Normal force from the bottom is related to the pressure at the bottom, since pressure is the force per area:

$$p = \frac{N}{A} \implies N = pA.$$

Because liquid is in equilibrium, the two forces must balance each other:

$$N = mg \implies pA = \rho A h g \implies p = \rho g h.$$

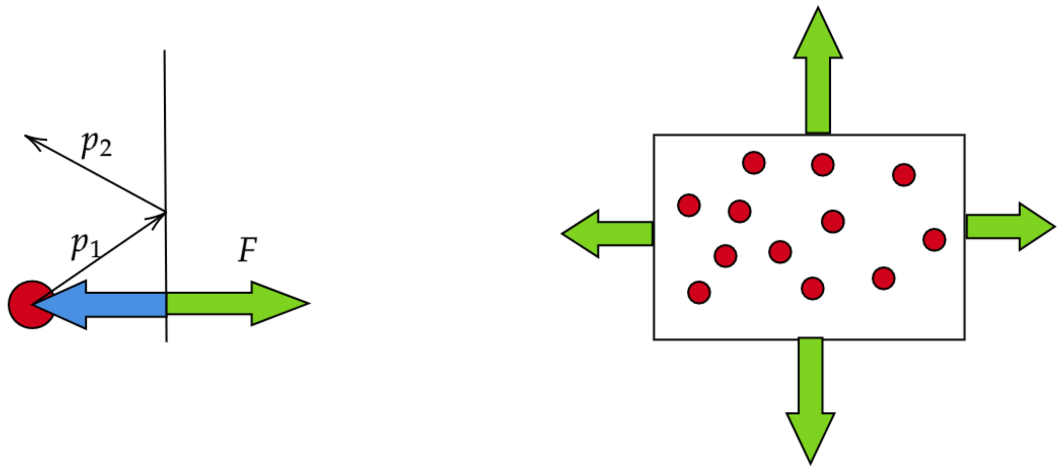

FIGURE 1. **Left figure:** if a molecule (red ball) collides with a container wall, molecule's momentum is changed from p_1 to p_2. Therefore wall acted on it with some force (shown in blue) and by Newton's third law the molecules acts on the wall with an equal force F (shown in green). **Right figure:** Force of pressure acting on a container wall is directed outwards on each wall. Note that magnitude of the force is larger for walls with bigger area: if pressure is the same, force grows with area.

The last formula is what we were looking for: it tells us how pressure grows with depth. For larger densities pressure grows faster. The cross section area canceled in the expression for pressure, so the formula is valid for a vessel of any cross-section (and, in fact, any shape, although we did not prove it here).

We have only calculated pressure that the liquid exerts on the bottom of the container. But it should push on the side walls as well: if there is a small opening in a container wall below the water level, water will start flowing out as a jet. The walls prevent the water from flowing out and experience water pressure as a result. So, fluids exert pressure in all directions and at a given point the pressure in all directions is equal.

Pressure in gases. Now let us to understand why a gas exerts pressure on the walls of its container. It is quite easy to understand from the microscopic picture of a gas. Remember that a gas consists of molecules flying around randomly and colliding with each other and the walls of the container. When a molecule hits a wall, its' momentum is changed which means the wall acted on it with some force. By Newton's third law it means that the molecule acted on the wall with some force. There is an enormous amount of molecules hitting the wall every second so the average force is very close to a constant and depends just on the area of the wall we are considering. Therefore, gas is characterized by its' pressure. The force with which the gas presses the wall is always directed perpendicularly to this wall and outwards from the container.

One example of pressure in a gas is atmospheric pressure. Our atmosphere acts upon every object with a pressure. At sea level it is normally about 100 kPa - so, on every square meter it acts with 100000 Newtons. How big is the atmospheric pressure? To appreciate its scale let's calculate the force with which atmosphere pushes down on the top surface of a desk. Let's say the desk has a rectangular shape, 1 meter by 2 meters. Since pressure is force divided by area, to find force we need to multiply pressure by area:

$$F = pA = 100000 \text{ Pa} \cdot 2 \text{ m}^2 = 200000 \text{ N}$$

How large is the force of 200000 N? Let's compare it to weight of something. An object with mass m has weight mg, so mass of the object with weight 200000 N is

$$m = \frac{200000 \text{ N}}{g} = \frac{200000}{10} \text{ kg} = 20000 \text{ kg}$$

This is a mass of 4 elephants! So the force atmospheric pressures acting on the top surface of the desk is the same as if we put there four elephants. However, the atmosphere does not break the desk because it not only presses on the surface from above, but it also presses on the bottom surface of the desk from below with exactly the same force (because pressure and area are the same). As a result the desk does not break, just squeezes a little.

Homework

1. A 45 kg skier has his ski on. The length of each ski is 1.5 m; the width is 10 cm. Find pressure that the skier is applying to the snow.

2. What pressure you produce when you are pushing a pushpin into a wall with a force of 50 N? Take the area of the pushpin tip as 0.01 mm^2. How does this pressure compare to the atmospheric pressure?

3. A fish tank 60 cm long, 40 cm wide and 30 cm high is full of water. Calculate pressure produced by the fish tank to the surface of the table. Water has density 1000 kg/m^3.

***4.** Estimate the mass of Earth's atmosphere. You are given atmospheric pressure $p_0 = 100000$ Pa and radius of the Earth $R = 6400$ km. *Hint: surface area of a sphere of radius R is $4\pi R^2$.*