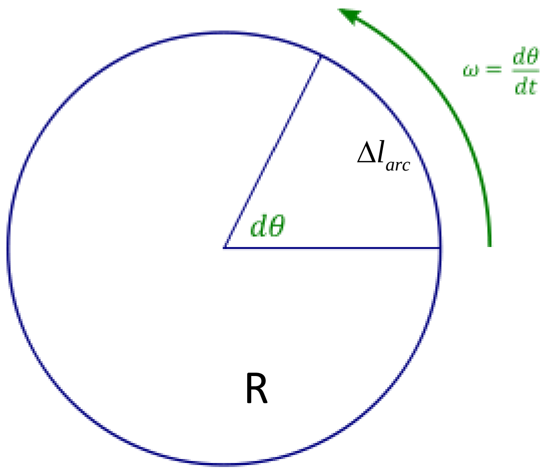
Rotation of a Solid Body

Angle (in radians): length of arc over radius

$$\Delta\theta = \frac{\Delta l}{R}$$



Angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

It is related to regular (linear) speed of rotational motion as:

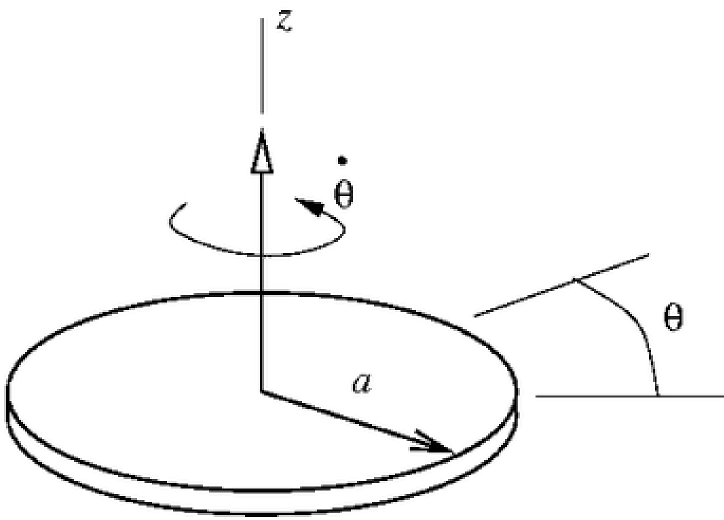
$$v = \frac{\Delta l_{arc}}{\Delta t} = \varpi R$$

Kinetic energy of a rotating object

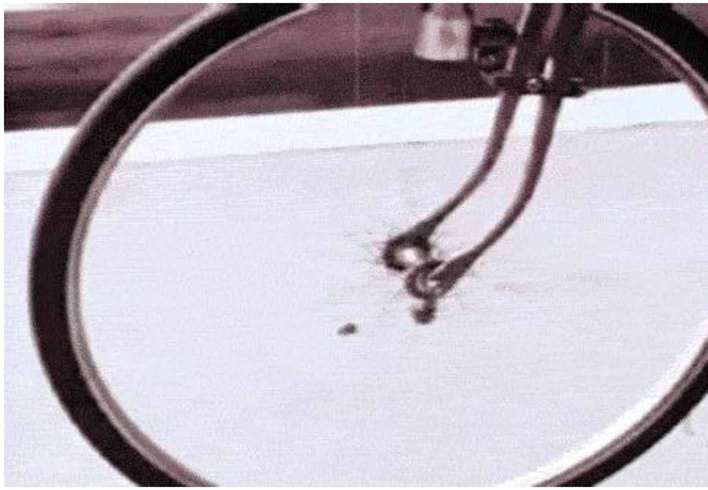
In a rotating rigid body, the further you are from the center, the larger is your speed!

Rotational kinetic energy is

$$K = \frac{I\omega^2}{2}$$



Here $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots.$
is called moment of inertia.
We pretended to "break" the solin object on many small pieces, m_n is its mass the n-th piece and r_n is its distance of from the axis of rotation.



You can easily find moment of inertia of a thin ring (or hoop, or bicycle wheel): most of its mass M is at the same distance R from the center, so

$$I_{ring} = MR^2$$

For other shapes, you can use reference tables

Homework

Problem

a) Moment of inertia of a uniform disk of mass M and radius R is $I_{disk} = MR^2/2$. Find the total kinetic energy of a disk as it rolls with linear speed V, without sliding. Note that its kinetic energy consists of regular (“translational”) and rotational one.

b) The disk starts rolling down a hill without sliding, with zero initial speed. What will be its final speed on the ground level, if the hill is 10 m high , and there is no energy loss?