MATH 8
HANDOUT 4: BINOMIAL THEOREM

Main formulas of combinatorics

Recall the numbers ${}_nC_k$ from Pascal's triangle:

$$\begin{aligned} {}_nC_k = & \text{ The number of paths on a chessboard going } k \text{ units up and } n-k \text{ units to the right} \\ & = \text{The number of words that can be written using } k \text{ zeros and } n-k \text{ ones} \\ & = \text{The number of ways to choose } k \text{ items out of } n \textbf{ if the order does not matter} \end{aligned}$$

We have discussed the following formula for them:

$$ {}_nC_k = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 1} = \frac{n!}{(n-k)!k!} \tag{1}$$

Binomial formula

These numbers have one more important application:

$$(a+b)^n = {}_nC_0a^n + {}_nC_1a^{n-1}b^1 + \cdots + {}_nC_nb^n \tag{2}$$

The general term in this formula looks like ${}_nC_k \cdot a^{n-k}b^k$. For example, for $n = 3$ we get

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

(compare with the 3rd row of Pascal's triangle)

This formula is called the **binomial formula**.

Problems

In all the problems, you can write your answer as a combination of factorials, ${}_nC_k$, and other arithmetic – you do not have to do the computations. As usual, please write your reasoning, not just the answers!

1. Use the binomial formula to expand the following expressions:
   (a) $(x-y)^3$
   (b) $(a+3b)^3$
   (c) $(2x+y)^5$
   (d) $(x+2y)^5$

2. Find the coefficient of x^8 in the expansion of $(2x+3)^{14}$

3. Solve AMC 8 2019