## MATH 6: HANDOUT XX <br> MATH BATTLE

1. The vertices of a triangle are $A(4,3), B(6,-1), C(-2,-5)$. $\mathrm{L}, \mathrm{M}$ are midpoints of BC and CA . Find the coordinates of L and M and show that $L M=\frac{1}{2} B A$
2. (a) Draw the graph of the equation $x^{2}+y^{2}-1=0$.
(b) Draw the graph of the equation $x^{2}+(y-1)^{2}-1=0$.
(c) Draw the graph of the equation $(x+2)^{2}+(y+3)^{2}=4$.
(d) Draw the graph of the equation $x y=0$.
3. Sketch the following function:

$$
f(x)=\left\{\begin{array}{l}
2 x+1 \quad \text { if } \quad x \leq 0 \\
-3 x^{2}+1 \quad \text { if } \quad x>0
\end{array}\right.
$$

4. (a) Find the equation for each of the lines shown in the graph below.
(b) Determine if two or more of the lines are perpendicular to each other, and prove it.

5. Solve the following systems of linear equations:

$$
\left\{\begin{array} { l } 
{ y = 3 x / 2 + 2 } \\
{ y + x = 2 }
\end{array} \left\{\begin{array} { l } 
{ y = 3 x / 2 + 2 } \\
{ x / 2 - y - 2 = 0 }
\end{array} \quad \left\{\begin{array}{l}
y=-x+2 \\
2 y-x=-4
\end{array}\right.\right.\right.
$$

6. Savir the Junior Hacker reprogrammed the elevator in the 100 -story Boogle Corporation building: only two buttons are currently working. The first button sends the elevator 8 floors up, and the second one 6 floors down. (The elevator will not move if it is asked to go above the 100th floor or below the 1st floor.)
(a) The company's CEO is currently drinking coffee on the first floor. (There is no lobby floor in the building). Can he take the elevator to the 95th floor? If so, show how. If not, explain why.
(b) Can he take the elevator to the 96th floor? If so, show how. If not, explain why.
7. (a) Write the equation for each of the circles and the line in the figure below.
(b) Find the two solutions for the following system of non-linear equations:

$$
\left\{\begin{array}{l}
y=-x+3 \\
4=(x+1)^{2}+(y-2)^{2}
\end{array}\right.
$$


8. Can a $5 \times 5$ chessboard be tiled by $2 \times 1$ dominoes? (to tile means to cover the boards completely, without any overlaps.) If so, show how. If not, explain why not.

