

**MATH 6: HANDOUT XII**  
**MATH BATTLE**

1. In a regular poker game, players are handed 5 cards each randomly from the deck. What is the probability that you get four of a kind? Four of a kind means that you get the same number from each kind. In poker, it does not matter the order in which you get the cards. Hint: First, determine in how many different ways you could make a four of a kind. Then, divide this by the total number of hands that could be dealt.
2. On the island of Knights and Knaves, you meet three inhabitants: Bob, Mel and Peggy. Bob says that it's not true that Peggy is a knave. Mel says that Peggy is a knight or Bob is a knave. Peggy claims, "Both I am a knight and Bob is a knave." Can you determine who is a knight and who is a knave?
3. I have three boxes. One is full of apples, another one is full of oranges, and the third one has a mixture of apples and oranges. The boxes are labelled "Apples", "Oranges", and "Mixed"; however, the three labels are all incorrect. You can choose any box to take one fruit out of it. Based on the result from that can you determine which box is which?
4. If you roll two dice:
  - (a) What is the probability that the sum of the numbers will be larger or equal than 9?
  - (b) What is the probability that the sum of the numbers will be smaller or equal than 5?
  - (c) What is the probability that the sum of numbers will be larger than five and smaller than 9?
5. Recall the logical operation  $\text{NAND}$ , which is defined by the following truth table:

$A$	$B$	$A \text{ NAND } B$
T	T	F
T	F	T
F	T	T
F	F	T

Show that any logical operation using  $\text{NOT}$ ,  $\text{AND}$ , or  $\text{OR}$  can be rewritten in terms of the  $\text{NAND}$  operation.

6. In a  $9 \times 9$  **checkered** board there is a pawn on each square. Can you move each pawn to one of the adjacent squares so that after the move, there is again one pawn in each square of the board? If so, show how to do it. If not, explain why not. Hint: Make a distinction between the pawns that are originally in white spaces from those that are originally in black spaces.
7. How many paths can you form on a grid with 3 rows and 4 columns if you start at the lower left corner and finish at the upper right corner? You can go only on the lines of the grid and you can only go to the right or up.
8. Imagine that there are 20 points in the plane. Out of the 20 point, three of them are colinear, meaning that there is a line that contains the three points. How many different triangles could you do by picking three points out of the 20 possibilities?