MATH 6: HANDOUT 27
LONG DIVISION

POLYNOMIAL DIVISION

In arithmetics, we divide numbers using long division. We can do the similar process for dividing polynomials.

For example, we can divide $2x^2 - 7x + 3$ by $x - 3$

$$\begin{array}{r|l} & 2x - 1 \\ \hline x - 3 & 2x^2 - 7x + 3 \\ & \underline{-2x^2 + 6x} \\ & -x + 3 \\ & \underline{x - 3} \\ & 0 \end{array}$$

This means that

$$2x^2 - 7x + 3 = (x - 3)(2x - 1)$$

Now, if we want to solve an equation

$$2x^2 - 7x + 3 = 0$$

we can equate both factors to 0, and get two solutions: when $x - 3 = 0$, then $x = 3$, and when $2x - 1 = 0$, then $x = 1/2$.

This can be useful if we want to solve an equation for which we don't have any good method. For example, if we are trying to solve an equation

$$2x^2 + 7x - 22 = 0$$

we can try to guess one of the solution using guess-and-check method. When we check small whole numbers, we can see that $x = 2$ is a solution, and that would mean that we can divide $2x^2 + 7x - 22$ by $x - 2$:

$$\begin{array}{r|l} & 2x + 11 \\ \hline x - 2 & 2x^2 + 7x - 22 \\ & \underline{-2x^2 + 4x} \\ & 11x - 22 \\ & \underline{-11x + 22} \\ & 0 \end{array}$$

This means that

$$2x^2 + 7x - 22 = (x - 2)(2x + 11)$$

and to solve this equation we have to equate two factors to 0: $x - 2 = 0$ gives us $x = 2$, and $2x + 11 = 0$ gives us $x = -5.5$.

Note that as with division of numbers, in some cases you may end up with a remander:

$$\begin{array}{r|l} & 2x + 15 \\ \hline x - 3 & 2x^2 + 9x - 22 \\ & \underline{-2x^2 + 6x} \\ & 15x - 22 \\ & \underline{-15x + 45} \\ & 23 \end{array}$$

HOMEWORK

1. Complete the long division of polynomials:
   (a) $x^2 - 3x - 4$ by $x - 4$
   (b) $x^3 - 2x^2 + 2x - 4$ by $x - 2$
   (c) $x^4 + 3x^3 - x^2 - x + 6$ by $x + 3$
   (d) $2x^4 - 5x^3 + 2x^2 + 5x - 10$ by $x - 2$

2. Solve the following equations by first guessing one of the solutions, and then using polynomial division.
   (a) $2x^2 + 3x - 14 = 0$
   (b) $3x^2 - 10x + 3 = 0$
   (c) $5x^2 + 8x - 4 = 0$

3. Solve the following inequalities:
   (a) $\frac{x}{x+1} > 2$
   (b) $(1 - x)(2x + 1) < 0$