Math 5b, homework 17.

1. Prove that the value of the following expressions is a rational number.

## Example:

$(\sqrt{3}-1)(\sqrt{3}+1)=\sqrt{3} \cdot \sqrt{3}+\sqrt{3} \cdot 1-1 \cdot \sqrt{3}-1=\sqrt{3} \cdot \sqrt{3}-1=(\sqrt{3})^{2}-1=$ $3-1=2$
a. $(\sqrt{7}-1)^{2}+(\sqrt{7}+1)^{2}$
b. $(\sqrt{7}-2)^{2}+4 \sqrt{7}$
2. Without using calculator compare:
$3 \ldots \sqrt{11}$
11 ... $\sqrt{110}$
$22 \ldots \sqrt{484}$
$5 \ldots \sqrt{20}$
$17 \ldots \sqrt{299}$
35 ... $\sqrt{1215}$
3. 6 painters can painters can paint the house in 5 days. How many painters are needed to do the job in 3 days?
4. John wrote the letters "J", "o", "h", and " $n$ " on 4 index cards. After that, he turned them blank side up, mixed them, aligned them, and turned them back to the "letter" side. How likely is it that $\square$
$\square$
$\square$ he will get his name written on the cards?
(Hint: Probability is the ratio of the number of ways the desired outcome can happen to all possible outcomes.)
5. Solve the following equations:
a. $2(x-1)=\frac{2}{3}(x+5)$;
b. $2 x-(5 x-7)=-1$
c. $\frac{x-2}{x-1}=3$
6. Do the following arithmetic operations with binary numbers. Do them without converting the numbers to decimal form.
a. $110101_{2}+111011_{2}$;
b. $10101_{2} \times 1011_{2}$;
c. $\left(10101_{2}+1101_{2}\right) \times 10110_{2}$
7. Simplify:
a. $\left(\frac{5 a^{2} b^{5}}{4 a^{3} b^{3}}\right)^{3}$;
b. $\left(2 z^{2} \cdot 3 z^{3} \cdot z\right)^{2}$;
c. $\frac{(-a b)^{8}}{(a b)^{2}}$;
d. $\left(\frac{3 a b^{3}}{15 b}\right)^{2} \cdot \frac{75 c}{a^{2} b^{6}}$;
e. $\left(\frac{3 a^{5} b^{2}}{21 a b}\right)^{2} \cdot \frac{7^{4}}{a^{16} b^{2}} ;$
f. $\frac{(-a b)^{8}}{(a b)^{2}}$

