

MATH 5: HANDOUT 18
CHOOSINGS AND PERMUTATIONS.

CHOOSING WITH REPETITIONS, REVIEW

Here are basic combinatorics laws in one place for your convenience:

- Multiplication rule: if there are k ways to choose the first item, and n ways to choose the second, then there are $k \times n$ ways to choose the pair
- If we need to choose k items, each of which can be selected from a list of n , and **order matters, repetitions are allowed**, then there are n^k ways to do this.
- If we need to choose k items, each of which can be selected from a list of n , and **order matters, repetitions are not allowed**, there are $n(n-1)\dots(n-k+1)$ ways of doing it (the product has k factors). This number is usually denoted

$${}_kP_n = n(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}.$$

Typical example: there are ${}_{10}P_{25}$ ways to seat 10 students in a room with 25 chairs.

- There are $k! = 1 \times 2 \times \dots \times k$ ways to order k items.
Typical example: there are $52!$ ways to shuffle a card deck.
- If we need to choose k items, each of which can be selected from a list of n , and **order does not matters, repetitions are not allowed**, then there are

$${}_kC_n = \frac{{}_kP_n}{k!} = \frac{n!}{k!(n-k)!}$$

ways to do this.

Typical examples: there are ${}_6C_{52}$ ways to choose six cards out of a deck of 52; if we toss a coin 10 times, there are ${}_4C_{10}$ combinations in which we have 4 heads and 6 tails.

Almost all combinatorial problems can be reduced to one of these.