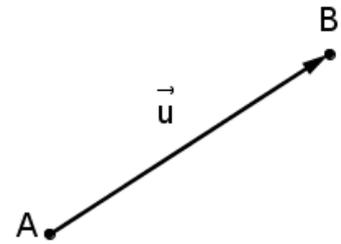


Translations

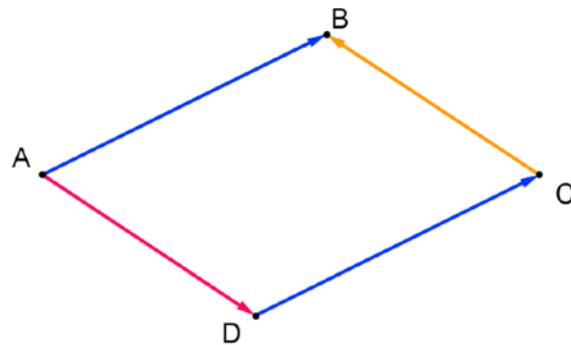
Definition. A **vector** (x,y) in the Cartesian plan is an « arrow » determined by its **length**, **direction** and its **orientation**.

Example. $\vec{u}=\vec{AB}$ is a vector of **origin/head** A et **tail** B . The **length/magnitude** of the vector is that of the **segment** $[AB]$, its **direction** is given by the line AB and its orientation is from A to B .



Attention. A vector is not a set of points so the vector \vec{AB} is not the segment $[AB]$.

Vector Equality Two vectors \vec{AB} , \vec{CD} are said to be equal if they have the same magnitude, orientation and direction, i.e. $ABCD$ is a **parallelogram**



Zero vector $\vec{0}=(0,0)$ is a vector of length 0.

Definition. A **translation vector** is a vector $\vec{u}=(x,y)$ that gives the length and direction of a particular translation of x units horizontally and y units vertically.

Construction of the image of a point through a translation of vector $\vec{u}=\vec{DE}$:

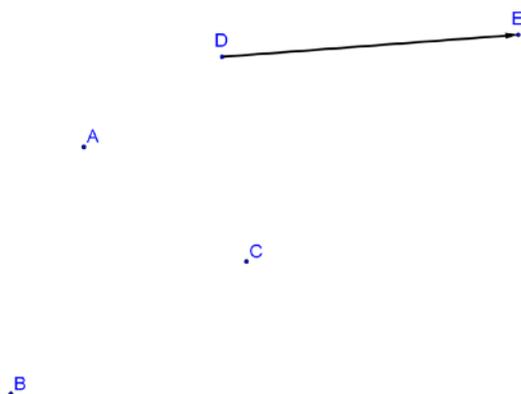


fig. 13

Construct on this image the translations of the points A, B, C through \vec{DE}

Does the translation t_{DE} have invariant points ?

.....

Does a translation have invariant points ?

.....
.....
.....

Observation:
Is there a translation such that **all the points of the Cartesian plan** are **invariants**?

Properties of a translation :
a) Collinearity. The image of a line

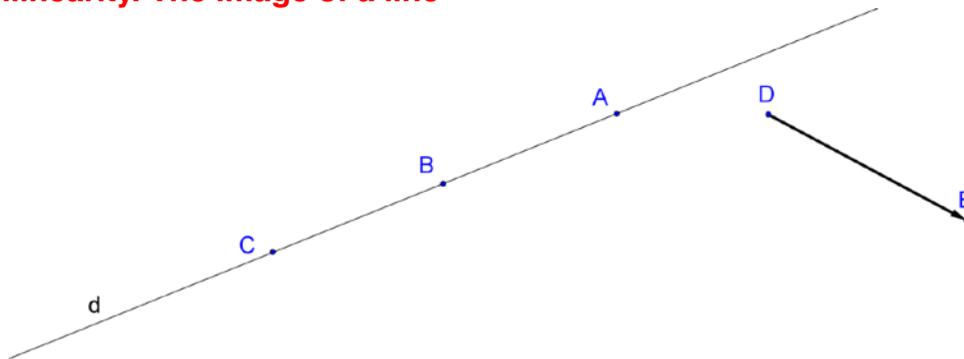


fig. 14

Construct the images A' , B' and C' of the points A , B et C through t_{DE}

.....
.....

Observation: A translation **conserves the collinearity of the points**.

What is the image of the line d through t_{DE} ? Compare the directions of the two lines!

.....
.....

Observation : The translation t_{DE} **transforms a line** d **into a line** with the line DE .

What are the **the invariant lines** through t_{DE} ?

.....
.....

b)Image of a segment

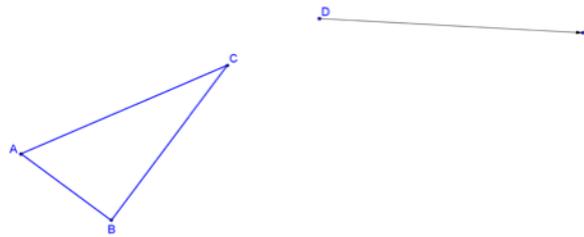


fig. 15

Construct the images of the segments , [AB], [BC] and [AC].

Can you write a general formula for the image of a point $A=(x,y)$ through a translation of vector $u=(h,k)$?

.....

Can you prove that if a line has the equation $Ax + By + C = 0$, the translated line has a new equation $Ax' + By' + (-Ah - Bk + C) = 0$? (Hint: plug the formulae for x' and y')

.....

.....

Observation: A translation **conserves distances**, so it is an **isometry** .

c) Conservation of angles

How are the angles of the triangle $A'B'C'$ with respect to the ones of ABC? Why?

.....

.....

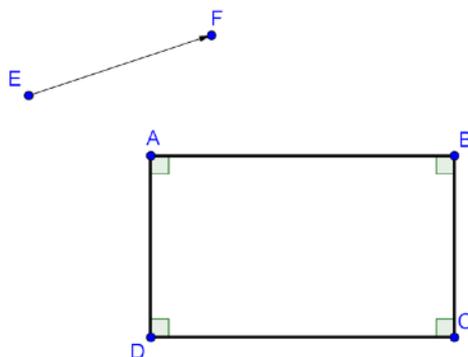


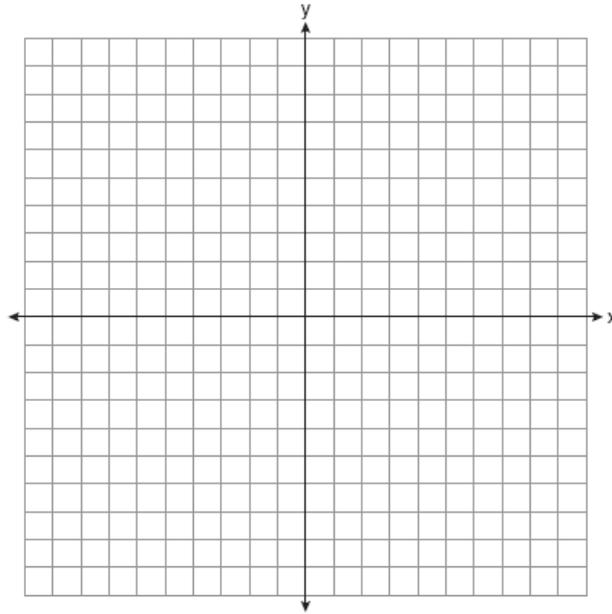
fig. 16

Observation :A translation is an isometry **conserving the orientation**.

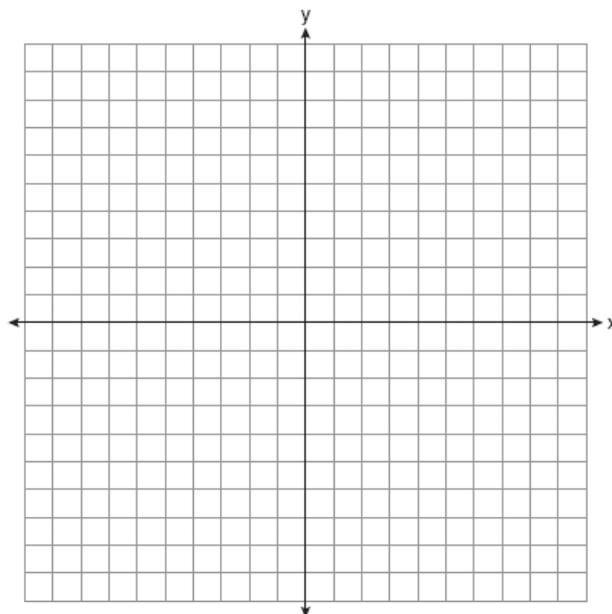
Exercises:

1. What is the image of the point $(-5, 2)$ under the translation $T_{3,-4}$?
2. When the transformation $T_{2,-1}$ is performed on point A , its image is point $A'(-3, 4)$. What are the coordinates of A ?
3. A translation moves $P(3, 5)$ to $P'(6, 1)$. What are the coordinates of the image of point $(-3, -5)$ under the same translation?
4. The image of point $(-2, 3)$ under translation T is $(3, -1)$. What is the image of point $(4, 2)$ under the same translation?
5. What is the image of the point $(-5, 2)$ under the translation $T_{3,-4}$?
6. When the transformation $T_{2,-1}$ is performed on point A , its image is point $A'(-3, 4)$. What are the coordinates of A ?
7. The image of $\triangle ABC$ under a translation is $\triangle A'B'C'$. Under this translation, $B(3, -2)$ maps onto $B'(1, -1)$. Using this translation, the coordinates of image A' are $(-2, 2)$. Determine and state the coordinates of point A .
8. If translation T maps point $(-3, 1)$ onto point $A'(5, 5)$, which is translation T ?
9. If the transformation $T_{(x,y)}$ maps point $A(1, -3)$ onto point $A'(-4, 8)$, what is the value of x ?
10. Translation T maps point $(2, 6)$ to point $(4, -1)$. What is the image of point $(-1, 3)$ under translation T ?
11. The coordinates of $\triangle JRB$ are $J(1, -2)$, $R(-3, 6)$, and $B(4, 5)$. What are the coordinates of the vertices of its image after the transformation $T_{2,-1} \circ r_{y\text{-axis}}$?
 - 1) $(3, 1), (-1, -7), (6, -6)$
 - 2) $(3, -3), (-1, 5), (6, 4)$
 - 3) $(1, -3), (5, 5), (-2, 4)$
 - 4) $(-1, -2), (3, 6), (-4, 5)$

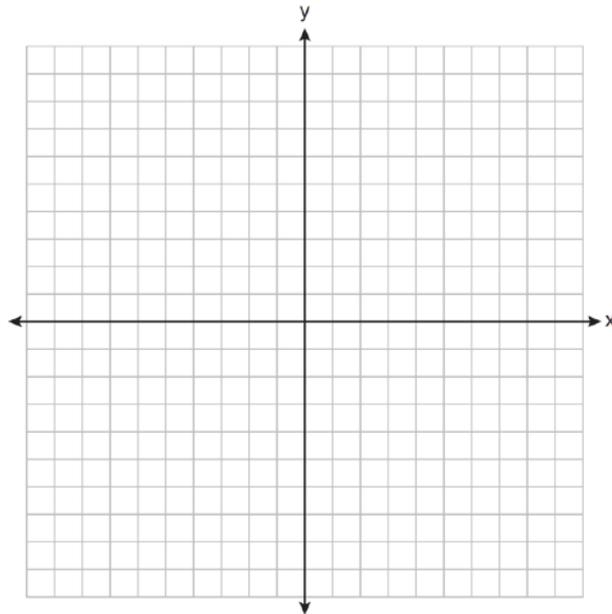
12. Quadrilateral *MATH* has coordinates $M(-6, -3)$, $A(-1, -3)$, $T(-2, -1)$, and $H(-4, -1)$. The image of quadrilateral *MATH* after the composition $r_{x\text{-axis}} \circ T_{7,5}$ is quadrilateral $M''A''T''H''$. State and label the coordinates of $M''A''T''H''$. [The use of the set of axes below is optional.]



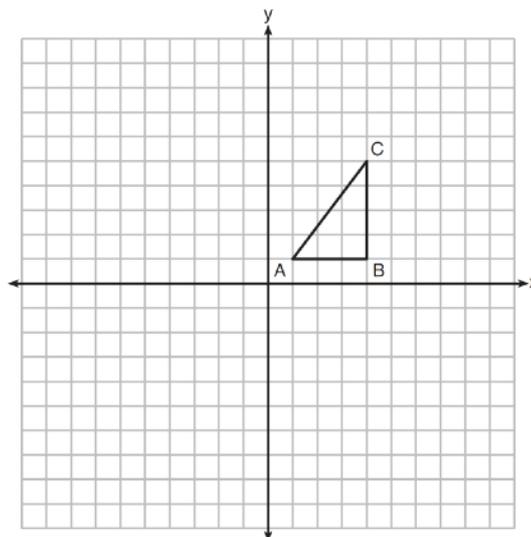
13. Quadrilateral *HYPE* has vertices $H(2, 3)$, $Y(1, 7)$, $P(-2, 7)$, and $E(-2, 4)$. State and label the coordinates of the vertices of $H''Y''P''E''$ after the composition of transformations $r_{x\text{-axis}} \circ T_{5,-3}$. [The use of the set of axes below is optional.]



14. The coordinates of the vertices of parallelogram $ABCD$ are $A(-2, 2)$, $B(3, 5)$, $C(4, 2)$, and $D(-1, -1)$. State the coordinates of the vertices of parallelogram $A''B''C''D''$ that result from the transformation $r_{y\text{-axis}} \circ T_{2,-3}$. [The use of the set of axes below is optional.]



15. In the diagram below, $\triangle ABC$ has coordinates $A(1, 1)$, $B(4, 1)$, and $C(4, 5)$. Graph and label $\triangle A''B''C''$, the image of $\triangle ABC$ after the translation five units to the right and two units up followed by the reflection over the line $y = 0$.



Homework

1. The image of point $(-2, 3)$ under translation T is $(3, -1)$. What is the image of point $(4, 2)$ under the same translation?

1) $(-1, 6)$

2) $(0, 7)$

3) $(5, 4)$

4) $(9, -2)$

2. The image of the origin under a certain translation is the point $(2, -6)$. The image of point $(-3, -2)$ under the same translation is the point

1) $(-6, 12)$

2) $(-5, 4)$

3) $\left(-\frac{3}{2}, \frac{1}{3}\right)$

4) $(-1, -8)$

3. Triangle ABC has vertices $A(1, 3)$, $B(0, 1)$, and $C(4, 0)$. Under a translation, A' , the image point of A , is located at $(4, 4)$. Under this same translation, point C' is located at

4. A design was constructed by using two rectangles $ABDC$ and $A'B'D'C'$. Rectangle $A'B'D'C'$ is the result of a translation of rectangle $ABDC$. The table of translations is shown below. Find the coordinates of points B and D' .

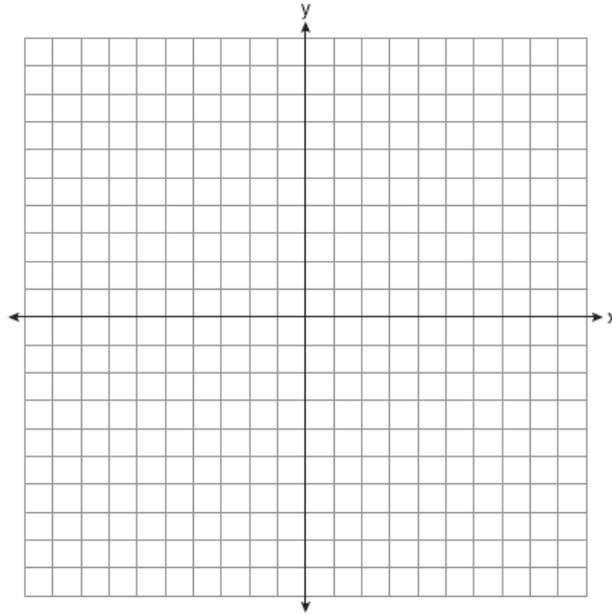
Rectangle $ABDC$	Rectangle $A'B'D'C'$
A $(2, 4)$	A' $(3, 1)$
B	B' $(-5, 1)$
C $(2, -1)$	C' $(3, -4)$
D $(-6, -1)$	D'

5. The image of point $(-2, 3)$ under translation T is $(3, -1)$. What is the image of point $(4, 2)$ under the same translation?

6. The image of the origin under a certain translation is the point $(2, -6)$. The image of point $(-3, -2)$ under the same translation is the point
7. Triangle ABC has vertices $A(1, 3)$, $B(0, 1)$, and $C(4, 0)$. Under a translation, A' , the image point of A , is located at $(4, 4)$. Under this same translation, point C' is located at
8. The image of $\triangle ABC$ under a translation is $\triangle A'B'C'$. Under this translation, $B(3, -2)$ maps onto $B'(1, -1)$. Using this translation, the coordinates of image A' are $(-2, 2)$. Determine and state the coordinates of point A .

A translation maps $P(3, -2)$ to $P'(1, 1)$. Under the same translation, find the coordinates of Q' , the image of $Q(-3, 2)$.

9. A translation maps $P(4, 1)$ to $P'(2, -1)$. What are the coordinates of Q' , the image of $Q(1, 3)$ under the same translation?
10. A translation maps $P(4, -3)$ onto $P'(0, 0)$. Find the coordinates of Q' , the image of $Q(-2, 1)$ under the same translation.
11. A translation maps the origin to the point $(5, -3)$. What is the image of the point $(-3, 2)$ under the same translation?
12. Under a given translation, the origin maps onto the point $(3, 5)$. What is the image of the point $(7, -1)$ under this same translation?
13. A translation maps the point $(5, -2)$ to a point $(0, -2)$. What is the image of the point $(0, -2)$ under the same translation?
14. A translation maps $(2, 1)$ onto $(-3, 2)$. Find the image of $(4, -1)$ under the same translation.
15. A translation maps $P(3, -2)$ onto $P'(5, 0)$. Find the coordinates of the image of $Q(4, -6)$ under the same translation.
16. A translation maps $A(-2, 1)$ onto $A'(2, 2)$. Find the coordinates of B' , the image of $B(-4, -5)$, under the same translation.
17. Find the coordinates of the image of $(2, 4)$ under the transformation $r_{y\text{-axis}} \circ T_{3,-5}$.
18. Find the image of point $A(3, -2)$ under the composition of translations $T_{2,1} \circ T_{-6,-4}$.
19. The vertices of $\triangle RST$ are $R(-6, 5)$, $S(-7, -2)$, and $T(1, 4)$. The image of $\triangle RST$ after the composition $T_{-2,3} \circ r_{y=x}$ is $\triangle R''S''T''$. State the coordinates of $\triangle R''S''T''$. [The use of the set of axes below is optional.]



20. The coordinates of the vertices of $\triangle ABC$ are $A(-6, 5)$, $B(-4, 8)$, and $C(1, 6)$. State and label the coordinates of the vertices of $\triangle A''B''C''$, the image of $\triangle ABC$ after the composition of transformations $T_{(4, -5)} \circ r_{y\text{-axis}}$. [The use of the set of axes below is optional.]

