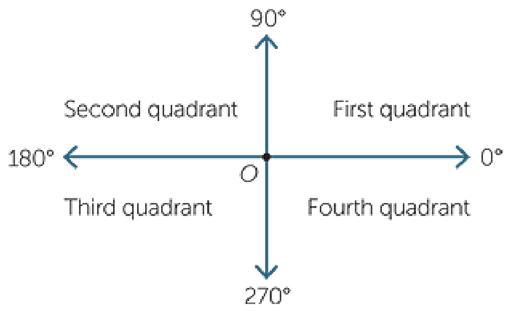
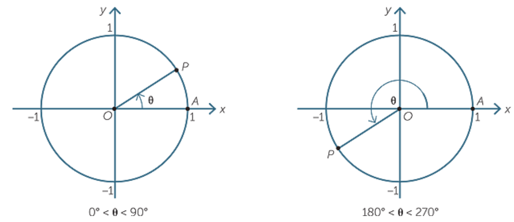




CHAPTER 7

TRIGONOMETRIC RATIOS CONTINUED

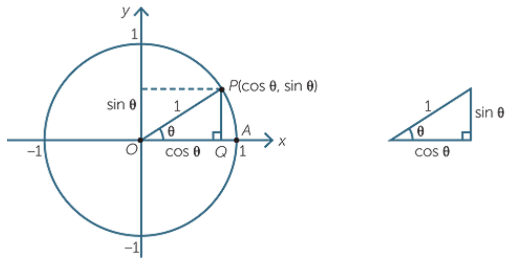
Trigonometric Functions						
Function	Notation	Definition	0	30	45	60
Sine	$sin(\alpha)$	$\frac{\text{opposite side}}{\text{hypotenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cosine	$cos(\alpha)$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
Tangent	$tan(\alpha)$	$\frac{\text{opposite side}}{\text{adjacent side}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

7.1 Unit Circle

The unit circle is divided into four quadrants corresponding the the quadrants of the XOY coordinate system. The angles are measured counterclockwise starting from the positive x-axis. Thus, in the first quadrant the angles measure between 0 and 90 degrees, in the 2-nd quadrant between 90 and 180 degrees, in the 3-nd quadrant between 180 and 270 degrees, in the 4-th quadrant between 270 and 360 degrees. We consider the angles measured in the clockwise sense to be negative. When using the unit circle formulation (or looking at trigonometric functions on the Cartesian Coordinate System) we usually use radian measure rather than degree measurement. Radians are simply another unit for measuring the size of an angle. To convert from degrees to radians and back use the circumference of a circle, $2\pi R$. For the unit circle it becomes 2π. So

$$2\pi \text{ radians} = 360 \rightarrow \pi \text{ radians} = 180$$

$$x \text{ degrees} = \frac{x}{180}\pi \text{ radians and } x \text{ radians} = 180\dot{x} \text{ degrees}$$



(a) Trigonometric Circle (b) Right Triangle

Correspondence : (x,y) points on the unit circle $(cos(\theta), sin(\theta))$

We take in the XOY Cartesian plane coordinate system the circle of radius 1, centered in the origin. We take a P(x,y) on the circle in the first quadrant. Drawing its x-coordinate, and y-coordinate we can construct a right-angled triangle with O at the origin. We will call θ the angle between the positive x axis and the hypotenuse.

Recall that

$$(x, y) = (cos(\theta), sin(\theta))$$

By definition

$$\cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{x}{1} \rightarrow x = \cos(\theta), \text{ and } \sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{y}{1} \rightarrow y = \sin(\theta)$$

7.2 Trigonometric identities

The trigonometric identities are very useful whenever you are simplifying or solving trigonometric expressions, or finding the measures of more angles. Most of the identities come directly from the Pythagorean Theorem, and a little algebra.
First,

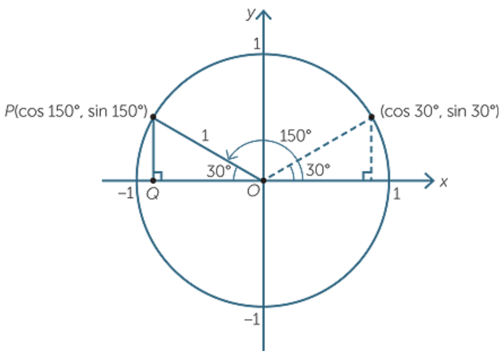
$$sin^2(\theta) + cos^2(\theta) = 1, \text{ for any angle } \theta$$

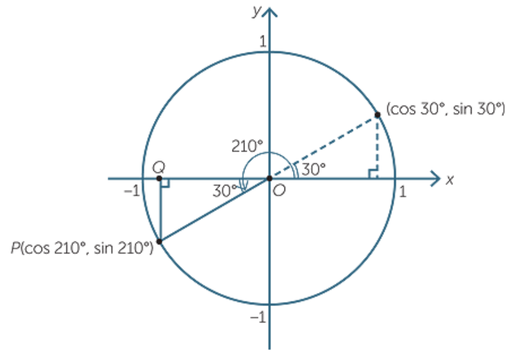
We just need to apply Pythagorean Th. in $\Delta OPQ : OP^2 = x^2 + y^2 \rightarrow 1 = cos^2(\theta) + sin^2(\theta)$
The other elementary trigonometric identity is

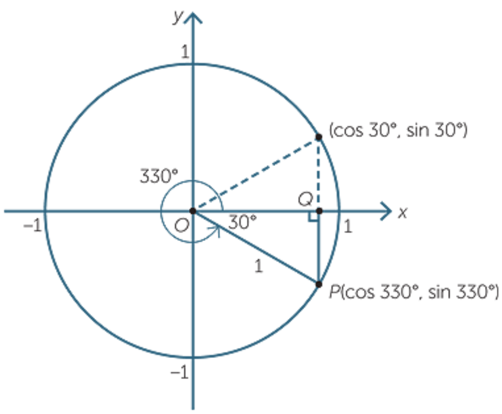
$$tan(\alpha) = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{opposite side}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{sin(\alpha)}{cos(\alpha)}$$

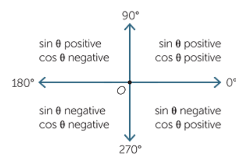
We also recall :

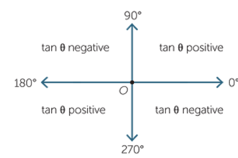
$$tan(\alpha) = m_{OP}, \text{ the tangent equals the slope of the hypotenuse OP}$$











7.3 The sine and the cosine from quadrant to quadrant

Let us take $\theta = 30$, and the point P on the circle of coordinates $(\cos(30), \sin(30))$.
Let us move point P around the circle until it arrives in the second quadrant and it makes an angle of 150 with the positive x-axis : $\sin(30) = \sin(150) = 0.5$ and $\cos(150) = -\cos(30) = -\frac{\sqrt{3}}{2}$. Let us move the point P around the circle until it arrives in the third quadrant and it makes an angle of 210 with the positive x-axis : $\sin(210) = -\sin(150) = -\sin(30) = -0.5$ and $\cos(210) = \cos(150) = -\cos(30) = -\frac{\sqrt{3}}{2}$. Let us move the point P around the circle until it arrives in the fourth quadrant and it makes an angle of 330 with the positive x-axis : $\sin(330) = -\sin(30) = -0.5$ and $\cos(330) = \cos(30) = \frac{\sqrt{3}}{2}$.
In general, we need to find for any θ its the acute reference angle $\theta - 180 > 0$, if $\theta > 180$ or $180 - \theta$ if $\theta < 180$

Table of sin (angle)

Angle	sin (a)	Angle	sin (a)	Angle	sin (a)	Angle	sin (a)
0.0	0.0	25.0	.4226	46.0	.7193	71.0	.9455
1.0	.0174	26.0	.4384	47.0	.7314	72.0	.9511
2.0	.0349	27.0	.4540	48.0	.7431	73.0	.9563
3.0	.0523	28.0	.4695	49.0	.7547	74.0	.9613
4.0	.0698	29.0	.4848	50.0	.7660	75.0	.9659
5.0	.0872	30.0	.5000	51.0	.7772	76.0	.9703
6.0	.1045	31.0	.5150	52.0	.7880	77.0	.9744
7.0	.1219	32.0	.5299	53.0	.7986	78.0	.9781
8.0	.1392	33.0	.5446	54.0	.8090	79.0	.9816
9.0	.1564	34.0	.5592	55.0	.8191	80.0	.9848
10.0	.1736	35.0	.5736	56.0	.8290	81.0	.9877
11.0	.1908	36.0	.5878	57.0	.8387	82.0	.9903
12.0	.2079	37.0	.6018	58.0	.8480	83.0	.9926
13.0	.2249	38.0	.6157	59.0	.8571	84.0	.9945
14.0	.2419	39.0	.6293	60.0	.8660	85.0	.9962
15.0	.2588	40.0	.6428	61.0	.8746	86.0	.9976
16.0	.2756	41.0	.6561	62.0	.8829	87.0	.9986
17.0	.2924	42.0	.6691	63.0	.8910	88.0	.9994
18.0	.3090	43.0	.6820	64.0	.8988	89.0	.9998
19.0	.3256	44.0	.6947	65.0	.9063	90.0	1.00
20.0	.3420	45.0	.7071	66.0	.9135		
21.0	.3584			67.0	.9205		
22.0	.3746			68.0	.9272		
23.0	.3907			69.0	.9336		
24.0	.4067			70.0	.9397		

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Table of cos(angle)

Angle	cos(a)	Angle	cos(a)	Angle	cos(a)	Angle	cos(a)
0.0	1.00	25.0	.9063	46.0	.6947	71.0	.3256
1.0	.9998	26.0	.8988	47.0	.6820	72.0	.3090
2.0	.9994	27.0	.8910	48.0	.6691	73.0	.2924
3.0	.9986	28.0	.8829	49.0	.6561	74.0	.2756
4.0	.9976	29.0	.8746	50.0	.6428	75.0	.2588
5.0	.9962	30.0	.8660	51.0	.6293	76.0	.2419
6.0	.9945	31.0	.8571	52.0	.6157	77.0	.2249
7.0	.9926	32.0	.8480	53.0	.6018	78.0	.2079
8.0	.9903	33.0	.8387	54.0	.5878	79.0	.1908
9.0	.9877	34.0	.8290	55.0	.5736	80.0	.1736
10.0	.9848	35.0	.8191	56.0	.5592	81.0	.1564
11.0	.9816	36.0	.8090	57.0	.5446	82.0	.1392
12.0	.9781	37.0	.7986	58.0	.5299	83.0	.1219
13.0	.9744	38.0	.7880	59.0	.5150	84.0	.1045
14.0	.9703	39.0	.7772	60.0	.5000	85.0	.0872
15.0	.9659	40.0	.7660	61.0	.4848	86.0	.0698
16.0	.9613	41.0	.7547	62.0	.4695	87.0	.0523
17.0	.9563	42.0	.7431	63.0	.4540	88.0	.0349
18.0	.9511	43.0	.7314	64.0	.4384	89.0	.0174
19.0	.9455	44.0	.7193	65.0	.4226	90.0	0.0
20.0	.9397	45.0	.7071	66.0	.4067		
21.0	.9336			67.0	.3907		
22.0	.9272			68.0	.3746		
23.0	.9205			69.0	.3584		
24.0	.9135			70.0	.3420		

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Table of tan(angle)

Angle	tan(a)	Angle	tan(a)	Angle	tan(a)	Angle	tan(a)
0.0	0.00	25.0	.4663	46.0	1.0355	71.0	2.9042
1.0	.0175	26.0	.4877	47.0	1.0724	72.0	3.0777
2.0	.0349	27.0	.5095	48.0	1.1106	73.0	3.2709
3.0	.0524	28.0	.5317	49.0	1.1504	74.0	3.4874
4.0	.0699	29.0	.5543	50.0	1.1918	75.0	3.7321
5.0	.0875	30.0	.5773	51.0	1.2349	76.0	4.0108
6.0	.1051	31.0	.6009	52.0	1.2799	77.0	4.3315
7.0	.1228	32.0	.6249	53.0	1.3270	78.0	4.7046
8.0	.1405	33.0	.6494	54.0	1.3764	79.0	5.1446
9.0	.1584	34.0	.6745	55.0	1.4281	80.0	5.6713
10.0	.1763	35.0	.7002	56.0	1.4826	81.0	6.3138
11.0	.1944	36.0	.7265	57.0	1.5399	82.0	7.1154
12.0	.2126	37.0	.7535	58.0	1.6003	83.0	8.1443
13.0	.2309	38.0	.7813	59.0	1.6643	84.0	9.5144
14.0	.2493	39.0	.8098	60.0	1.7321	85.0	11.430
15.0	.2679	40.0	.8391	61.0	1.8040	86.0	14.301
16.0	.2867	41.0	.8693	62.0	1.8907	87.0	19.081
17.0	.3057	42.0	.9004	63.0	1.9626	88.0	28.636
18.0	.3249	43.0	.9325	64.0	2.0503	89.0	57.290
19.0	.3443	44.0	.9657	65.0	2.1445	90.0	infinite
20.0	.3640	45.0	1.000	66.0	2.2460		
21.0	.3839			67.0	2.3559		
22.0	.4040			68.0	2.4751		
23.0	.4245			69.0	2.6051		
24.0	.4452			70.0	2.7475		

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Source of drawings : The International Centre of Excellence for Education in Mathematics (ICE-EM), Source of Trigonometric Tables : www.grc.nasa.gov,

7.4 Problems

1. Use the reference angle θ to determine sin(195), sin(210), cos(120).
   [195 is in the 3-rd quadrant $195 \geq 180$ then 195-180=15 and $\sin(190) = -\sin(15) = -.2528$]

2. Using angles from all the four quadrants, write all the expressions equivalent to cos(120).

3. Use the reference angle θ to determine $\cos(-120)$.
   [$\cos(-120) = \cos(-120+360) = \cos(240) = -\cos(240-180) = -\cos 60$ (240 is in the 3-rd quadrant)]

4. Write all the sine and cosine values equal to sin(180).

5. For which values from 0 to 360 is $\tan(\theta)$ undefined ?

6. Draw on the trigonometric circle the angle 60 and find the coordinates of the point $P(\cos(60), \sin(60))$.

7. Draw on the trigonometric circle the angle 240 and find the coordinates of the point $P(\cos(240), \sin(240))$.

8. Evaluate the expressions $\sin^2(\theta) + \cos^2(\theta)$ and $\sin^2(2\theta) + \cos^2(2\theta)$

9. Simplify the expression $\frac{\cos^2(\theta)}{tan(\theta)}$

10. Simplify the expression $\frac{1-\sin^2(\theta)}{cos^2(\theta)}$

11. Simplify the expression $\frac{sin(\theta)}{cos(\theta)\cdot\tan(\theta)}$