CHAPTER 6

BASIC TRIGONOMETRIC RATIOS

Discovery homework

A. Use graph paper, a ruler and a protractor:

- Draw a 1cm (or 1in, according to your ruler) segment AB, and then a perpendicular line h at one of its ends (say A).
- Draw a line d from the other end (so from B), at 30° from AB, until it intersects the perpendicular line, in C.

1. Now, that we have a right triangle ΔABC with the right angle in A and with $\widehat{ABC}$ of 30°, we can measure the lengths of AC and BC, and compute $\dfrac{AC}{BC}$. What value do you find for it ?
2. Repeat the process for another segment AB twice as long, then three times as long – what values do you keep finding ?
3. What is the value of $\dfrac{AB}{BC}$ for each case ?
4. Repeat the whole process for an angle of 45°: how would AC and AB be related, what values do you find for these fractions ?

B. Use graph paper, a ruler, and a compass (the drawing tool to trace circles or arcs):

- Set a point as origin of the coordinate plane and draw the Ox and Oy axes (horizontal and vertical, with arrows, and label them), and draw a vertical segment from $A(1,1)$ to $B(1,3)$.
- Draw a circle centered on B with radius 2 (passing through A as well).
- Find the middle P of AB.
- Draw a horizontal line from P, intersecting the circle in a new point M
- Draw a horizontal line d from A
- Draw the line BM until it intersects the line d in a new point C.
- Notice that M must be the middle of BC (because P is the middle of AB and PM is parallel to AC)
- Notice that $BM = AB$ (since M and A belong to the same circle centered in B).

1. Compute $\dfrac{AB}{BC}$.
2. What kind of triangle is ΔABC ?

3. What kind of triangle is ΔPBM ?
4. Compute $\dfrac{BP}{BM}$.
5. If we tell you that also $AM = \frac{BC}{2}$, can you find out how many degrees the angle $\widehat{ACB}$ has ? (Hint: think of what kind the triangle ΔABM is, and then go around subtracting angles, since you know what triangle ΔABC is).
6. How many degrees does the angle $\widehat{BMP}$ have ?
7. How long must AC be (from Pythagora's) ?
8. Compute $\dfrac{AC}{BC}$.
9. Compute $\dfrac{PM}{BM}$.
10. Take a graduated ruler and measure AC, BC, PM and BM and confirm that they are reasonably close to what you computed. You can also take a protractor and do the same for the angles.

C. If you did A. and B. and want to have more fun: use graph paper, a ruler, a compass and a protractor.

- Turn the page sideways (in "landscape" mode), draw a straight horizontal line through its middle (our Ox axis), and a fairly large circle centered in the left half.
- Draw a vertical Oy axis not too far from the circle, to its right, on one of the graph paper existing grid lines.
- Divide the circle into twenty slices (so each of them has an angle of $18°$) at the center of the circle). Consider the first slice the one to the right, with one side on the big horizontal line Ox, the second slice the one above the first (thus anti-clockwise), the third slice above the second and so on (so remember this numbering).
- Extend dashed horizontal lines from each point on the circle corresponding to these slices all the way to the right end of the page
- From the O on the xOy, mark twenty equal divisions on the Ox axis, to its positive direction (align them on the grid, to help, so basically draw twenty thin vertical dashed lines)
- Mark the intersections of each of these vertical lines with the corresponding horizontal lines: the first horizontal (from the first slice) with the first vertical (the closest to O), the second horizontal with the second vertical (the one right to the first vertical) and so on.
- So, what we are doing is plotting points of coordinates (x, y) where x=some angle from the horizontal, measured anti-clockwise, and y= the height where the radius of that angle meets its circle. Check that you are obtaining a wavy pattern, starting at O, and rising up to the height equal to the radius of the circle, then going back to zero, then below to negative of the radius, and back to zero. Those are points of the sine function. This functions goes on and on, both left and right of the origin, since the slicing can continue on and on, as "traces" of a watch handle for instance.

Question: For each slice, we can talk about the right triangle with hypothenuse the radius and one side the segment on the Ox. What if we were to use the length of that side (on Ox, that is), as y-coordinate when plotting points ? Can you find a geometrical reason why it would "look the same" (that is also wavy, also up and down between radius and negative radius, etc) ? Hint: think of repeating the same process as above, but with a new (thus different) Ox axis, this time vertical (and of course still passing through the center of the circle) – how would that unfold ?