

## Math 6d: Homework 4

HW#4 is Due October 21; submit to Google classroom 15 minutes before the class time.

*Please, write clearly which problem you are solving and show all steps of your solution.*

**Classical logic** is the study of arguments. Argument is a sequence of statements of which one is a conclusion. The other statements (premises) give evidence or information and help to reach a conclusion. Classical logic evaluates statements as only True or False. The evaluation as True or False depends on who is evaluating the statement.

**Logic variables** are basic statements labeled with A, B, C, ..., that can be evaluated as T or F.

For example,  $A = \text{"Bill is a knight"}$  and  $C = \text{"Carl is a knight"}$  can be evaluated (answered) as True or False.

**Logic operations** AND, OR, NOT combine variables (statements) in more complicated statements. A **truth table** for a logic operation evaluates all possible combinations of two variables that are combined by the logic operation.

Examples: Operation AND:  $A \text{ AND } B = \text{"Bill is a knight AND Carl is a knight"}$

Operation OR:  $A \text{ OR } B = \text{"Bill is a knight OR Carl is a knight"}$ ,

Operations NOT:  $\text{NOT } A = \text{"Bill is not a knight"}$ .

Truth table AND

A	B	A AND B
T	T	T
T	F	F
F	T	F
F	F	F

Truth table OR

A	B	A OR B
T	T	T
T	F	T
F	T	T
F	F	F

Truth table NOT

A	NOT A
T	F
F	T

Some of the problems in this assignment are from the island of The Knights and Knaves. Remember, Knights always tell the truth, and Knaves always lie. You can find these problems and many more at:

<http://philosophy.hku.hk/think/logic/knights.php>

## Homework questions

1. Solve the following equations:

(a)  $2x - 22 = 3(1 - x)$

(b)  $1 - \frac{2}{7}x = \frac{1}{7}x$

(c)  $1 - 8(1 - x) = 7x - 8$

2. At the minimum, write the statements made by the inhabitants using logic variables and operations. Then try to solve the problem.

Variables:  $A = \text{"Bob is a Knight"}$ ,  $B = \text{"Mel is a Knight"}$ ,  $C = \text{"Peggy is a Knight"}$ .

On the island of Knights and Knaves, you meet three inhabitants: Bob, Mel, and Peggy. Bob says that it's not true that Peggy is a knave." Mel says that "Peggy is a knight or Bob is a knave." Peggy claims "both I am a knight and Bob is a knave."

3. Many trucks carry the message: “If you do not see my mirrors, then I do not see you.” Can you rewrite this in an equivalent form, without using the word ‘not’?
4. Define a new logical operation *XOR* (exclusive *OR*) as follows:  $A \text{ XOR } B$  is *only* true if exactly  $A$  is true or  $B$  is true, but not when both are true.

(a) Write the truth table for  $A \text{ XOR } B$

(b) Can you express *XOR* using only *AND*, *OR*, and *NOT* (that is, write a formula equivalent to  $A \text{ XOR } B$  using only *AND*, *OR*, and *NOT*)?

5. (a) Write truth tables for formulas  $A \text{ AND } (B \text{ OR } C)$  and  $(A \text{ AND } B) \text{ OR } C$  (hint: there will be 8 rows in the table). Are these formulas equivalent? (i.e. do they always give the same answer?)

(b) The waiter in a restaurant tells you: “our fixed price dinner includes soup and appetizer or salad.”

Denoting

$A$  = your dinner will include soup

$B$  = your dinner will include appetizer

$C$  = your dinner will include salad

What would be the correct way to write his statement using letters  $A$ ,  $B$ ,  $C$  and logical operators *AND*, *OR*?

6. On the island next to the island of knights and knaves there are three kinds of people:

Knights, who always tell the truth,

Knaves, who always lie,

Normal people, who sometimes lie and sometimes tell the truth.

On that island, you meet 3 people,  $A$ ,  $B$ , and  $C$ , one of whom is a knight, one a knave, and one normal (but not necessarily in that order). They make the following statements:

$A$ : “I am normal”

$B$ : “That is true”

$C$ : “I am not normal”

What are  $A$ ,  $B$ , and  $C$ ? (Try to solve by using a truth table for the variables  $A$ ,  $B$ ,  $C$  as we did in class. If not, use the method you understand best.)