MATH 5: HANDOUT 13
MORE POWERS. SCIENTIFIC NOTATION.

More Powers

Recall that for a positive integer n, we have defined

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ times}}$$

then

$$a^m a^n = a^{m+n}, \qquad a^m \div a^n = a^{m-n}$$

It turns out that there is only one way to define a^n for $n = 0$ and negative n so that these rules still work, namely:

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

For example, $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Another important formula is the following:

$$(a^n)^m = a^{n \times m}.$$

It is easy to see why this formula holds:

$$(a^n)^m = \underbrace{(\underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ times}}) \times \cdots \times (\underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ times}})}_{m \text{ times}} = a^{n \times m}$$

Scientific notation

Scientific notation is a convenient way to write very large numbers: instead of writing $2,000,000,000$ one can say "2 and then 9 zeros". Since writing a zero at the end is the same as multiplication by 10, we can also write the same number as

$$2 \times 10 \times \cdots \times 10 \qquad \text{(9 times)}$$

or, for short 2×10^9. Thus, we can write

$$2,000,000,000 = 2 \times 10^9$$

which is much shorter.

Similarly, we can write

$$\begin{aligned} 2,310,000,000 &= 231 \times 10 \times \cdots \times 10 \qquad \text{(7 times)} \\ &= 2.31 \times 10 \times \cdots \times 10 \qquad \text{(9 times)} \\ &= 2.31 \times 10^9 \end{aligned}$$

Such a form (a decimal with one digit before decimal point times 10 to some power) is called the *scientific notation*.

To write a number larger than 10 in scientific notation, you should:

1. Count how many digits the whole part has. The power of 10 will be number of digits minus 1.
2. Write down the digits of the number, but now put the decimal point after the first digit.

Example:

$$3412000 = 3.412000 \times 10^6 = 3.412 \times 10^6$$

In a similar way, scientific notation is very useful for very small numbers. For example, weight of one atom of hydrogen is about 1.66×10^{-24} gram — or

$$0.00000000000000000000000166\ gr$$

CLASSWORK

1. Simplify:

(a) $(2z^2 \cdot 3z^3 \cdot z)^2$

(b) $\left(\dfrac{5g^4b^5}{4g^2b^3}\right)^3$

(c) $2x^2 \cdot x^3 - x^7 \div x^2$

(d) $\dfrac{(-ab)^8}{(ab)^2}$

(e) $\dfrac{18^{n+3}}{3^{2n+5} \cdot 2^{n-2}}$

(f) $\left(\dfrac{3ab^3}{15b}\right)^2 \cdot \dfrac{75c}{a^2b^6}$

2. Let $x = a^3 \cdot b^2$, $y = \dfrac{b^5}{a^2c^4}$, and $z = \dfrac{c^3}{ab}$. Express in terms of a, b, c:

(a) xyz

(b) $x^2y^3z^4$

(c) $\dfrac{xy}{z}$

HOMEWORK

1. If $a = 2^{-13}3^9$, $b = 2^{11}3^{-7}$, what is the value of ab? of a/b?

2. In how many zeroes does the number $4^{15}5^{26}$ end?

3. Simplify:

(a) $(4c^2 \cdot c^3)^3$

(b) $\left(\dfrac{8dg^2}{3d^3g^4}\right)^3$

(c) $((x^2y)^3)^4$

(d) $\dfrac{26(a^2b)^4}{65a^3b^2c^3}$

(e) $\left(\dfrac{9a^7b^6}{45a^3b}\right)^4$

(f) $\left(\dfrac{3a^5b^2}{21ab}\right)^4 \cdot \dfrac{7^4}{a^{16}b^2}$

4. Let $x = a^3 \cdot b^2$, $y = \dfrac{b^5}{a^2c^4}$, and $z = \dfrac{c^3}{ab}$. Express in terms of a, b, c:

(a) $(xy)^2z$

(b) $\dfrac{x}{y}$

(c) $\dfrac{x^3y^2}{xy^2z^3}$

5. Suppose $100 is deposited into an account and the amount doubles every 8 years. How much will be in the account after 40 years? Express your answer using powers.

6. At the beginning of an epidemic, 50 people are sick. If the number of sick people triples every other day, how many people will be sick at the end of 2 weeks? Express your answer using powers.

7. About how many hydrogen atoms are there in one gram of hydrogen?

8. Write the following numbers using scientific notation.
(a) the distance from Earth to Pluto is $\approx$ 7,527,000,000 km;
(b) the distance from Earth to the star Sirius is $\approx$ 81,900,000,000,000 km;
(c) the distance from Earth to Vega is $\approx$ 249,500,000,000,000 km;
(d) the distance from Earth to the Andromeda Nebula is $\approx$ 2,000,000,000,000,000,000,000 km.
(e) the area of the Pacific Ocean is $\approx$ 178,684,000,000 km^2

9. Write the following numbers in regular form:

(a) $9.21 \times 10^6 =$

(b) $1.527 \times 10^4 =$

(c) $5.3459 \times 10^3 =$

(d) $7.527 \times 10^2 =$