

## Homework 6.

### Torque

During last class we discussed torque. For rotational dynamics torque plays same role as force plays for the linear dynamics. If a nonzero torque applied to an object, the object experience angular acceleration  $\beta$ . Simply speaking, to turn an object we have to apply torque to it.

Imagine that you are trying to rotate a rigid body (Fig. 1) with respect to the axis passing through the point A (“pivot point”) and perpendicular to the page plane.

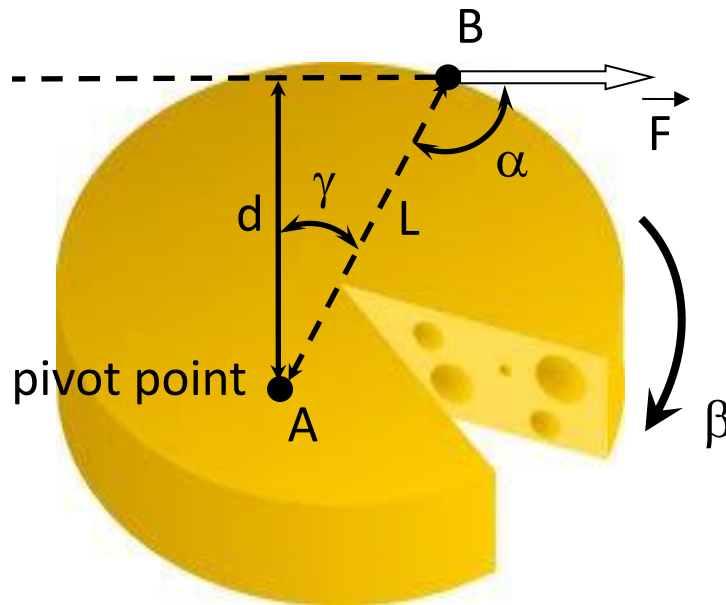


Figure 1.

$$|\vec{T}| = d \cdot |\vec{F}| \quad (1)$$

Torque can be calculated as a product of the force magnitude and the distance  $d$  between the pivot point and the straight line which passes through the point to which the force is applied (point B in Fig.1) and parallel to the force direction. Torque can also be calculated through the distance  $L$  between pivot point and the point of the force application. As we can see from the Figure 1,  $d = L \cdot \cos \gamma = L \cdot \sin \alpha$ . So

$$|\vec{T}| = |\vec{F}| \cdot L \cdot \sin \alpha \quad (2),$$

where  $\alpha$  is the angle between the force direction and the line connecting pivot point and point to which the force is applied. Torque is a vector – in a sense that it has both magnitude and direction. Let us consider the sign of the torque as the indication where it “wants” to turn the object – clockwise or counterclockwise. If we know total net torque  $T_{tot}$  which applied to the body, we can calculate the angular acceleration  $\beta$ :

$$\vec{T}_{tot} = I \cdot \vec{\beta} \quad (3),$$

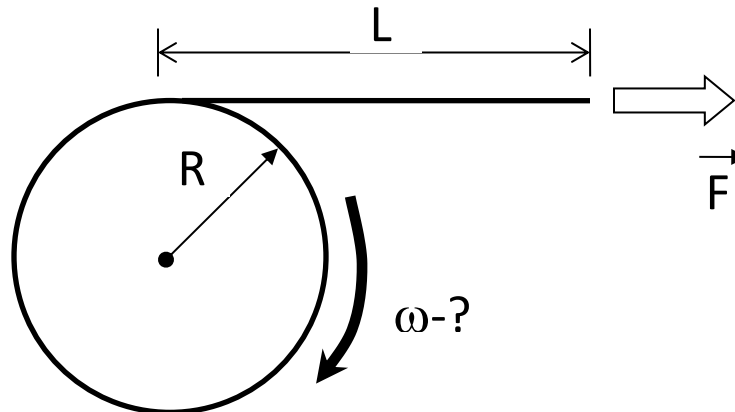
where  $I$  is the moment of inertia. Please note that both torque and the moment of inertia in the formula (3) should be calculated with respect to same axis. If you use a different axis both torque and moment of inertia can be different. Compare formula (3) with the Newton's second law:

$$\vec{F}_{tot} = m \cdot \vec{a} \quad (4),$$

Where  $m$  is mass,  $a$  – acceleration,  $F_{tot}$  – total net force applied to the body.

Problems:

1. A cylindrical coil of a radius  $R$  with thread can rotate without friction around the coil's "main" axes. You are pulling the thread with a constant force  $F$ . The mass of the coil is  $m$  (the thread is very light). What is angular velocity of the coil after you pulled out a thread's tail of the length  $l$ ? (See picture below).



2. Find final kinetic energy of the coil.