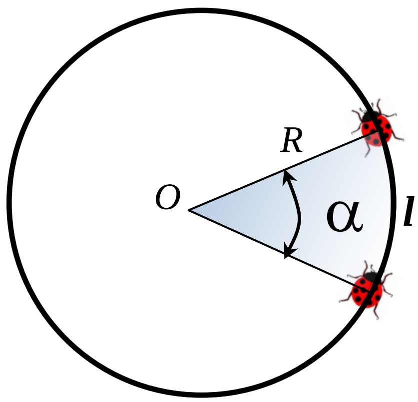
Homework 3

During last class we (hopefully) understood the relation between linear and angular speeds for circular motion. It is not difficult to express the angular speed through the linear speed using our "natural" way of angle measurement.

According to the definition, angular speed is the angle "swept" by a revolving object per unit time. (Just to be specific: we are using the word "rotation" if the axis passes through the rotating object. In case the axis is outside the object we will use the word "revolution"). Let us assume that the ladybug performs uniform revolution around axis *O* and sweeps angle $\boldsymbol{\alpha}$ during the time ***t***.



We can write the angular speed ω as:

$$\omega = \frac{\alpha}{t} \quad (1)$$

According to our definition of angle magnitude, the angle $\boldsymbol{\alpha}$ equals to the length of the circular arc ***l*** divided by the radius of the circle *R*.

$$\alpha = \frac{l}{R} \quad (2)$$

If we replace the angle $\boldsymbol{\alpha}$ in the expression 1 for ***l/R***, we will obtain:

$$\omega = \frac{l}{R \cdot t} \quad (3)$$

But the length of the arc ***l*** is the distance, passed by the ladybug during the time ***t***. So ***l/t*** is the linear speed ***v*** of the ladybug. Taking this into account we can write (3) as:

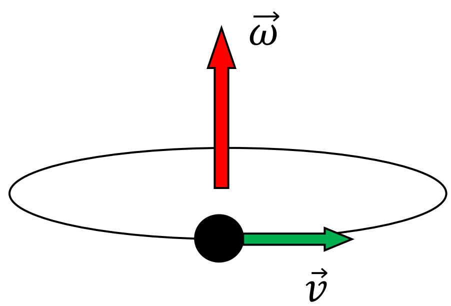
$$\omega = \frac{v}{R} \quad (4)$$

This is the expression, connection linear and angular speeds of the revolving object. We can see and the smaller radius at the same linear speed, the higher angular speed.

The time *T* which is required to complete one full turn is called the *period* of revolution. .Since in this case the angle passed is 2π,

$$T = \frac{2\pi}{\omega} \quad (5).$$

We have been discussing angular speed which does not have direction. What direction should we choose for the angular velocity? There are two ways for an object to move along a circular path with respect to us: clockwise and counterclockwise (these "directions" are swapped if we will be looking from the other side of the plane of the orbit). The vector of angular velocity is parallel to the axis of revolution (which means that it is perpendicular to the orbit plane). If we are "looking" from the end of the angular velocity vector, we will see that the object is revolving counterclockwise.



There is a convenient way to remember the relation between the directions of linear and angular velocities. It is called "right-hand rule":

Even if the angular velocity does not change, the motion is accelerated and acceleration with the magnitude of $\omega^2R=V^2/R$ is directed toward the center of the circular trajectory (centripetal acceleration). The angular velocity can change as well, both in magnitude and in direction. (we will not consider the latter case here).

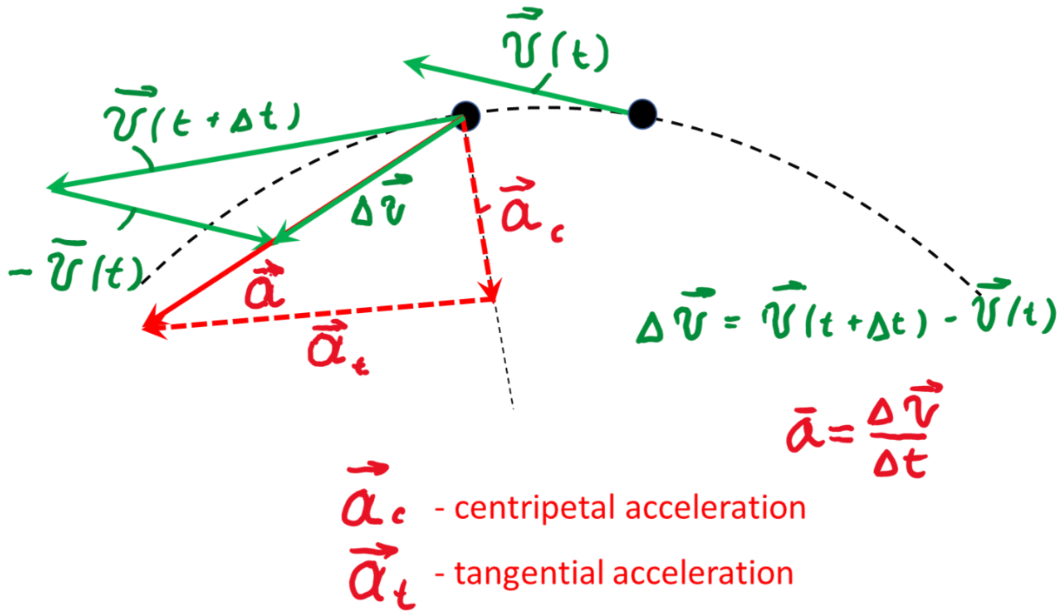


Figure 2. Centripetal and tangential acceleration.

As the magnitude of the angular velocity increases, the object revolves faster and faster, which means that its linear velocity increases as well. In this case we have a component of the acceleration which is directed along the velocity, i.e. along the tangent line to the trajectory. This acceleration we will call ***tangential*** acceleration.

Problem:

1. A driver moving in the car suddenly sees a fence across the road. What is the best strategy to avoid collision -to press the brake as hard as possible or to turn without pressing the brake? (Imagine that you have just these two options. Also, let us assume that traction control system of the car allows making a turn with the smallest possible radius, determined by the friction between the car's tires and the road). Explain your choice.