PRESSURE IN LIQUIDS.

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Theory recap

Today we start discussing the physics of liquids. We will be interested in equilibrium conditions in a fluid, so we will consider a stationary liquid, which does not move or flow. At every point liquid has some pressure, similarly to a gas. However, in a liquid pressure varies with height more significantly than in a gas (as we will see below it is because liquid is much more dense than gas). Let us derive what this variation is.

Consider a cylindrical vessel with liquid. Let the cross section area of the vessel be A, height of the liquid level be h, and density (mass per unit volume) of the liquid be ρ. Further let us assume that there is no atmospheric pressure (we will add it to consideration a bit later). We are interested in finding pressure of the liquid at the bottom of the vessel. In order to do it let us consider all forces acting on the liquid. There are two forces: gravity force mg and normal force from the bottom of the container N. Mass of the liquid is

$$m = \rho V = \rho A h$$

where V is volume of liquid, equal to product of cross section area and height. Normal force from the bottom is relate to the pressure at the bottom, since

$$p = \frac{N}{A} \implies N = pA.$$

Because liquid is in equilibrium, forces must balance each other:

$$N = mg \implies pA = \rho A h g \implies p = \rho g h.$$

The last formula is what we were looking for: it tells us how pressure grows with depth. For larger densities pressure grows faster. Note that cross section area canceled in the expression for pressure.

What happens if there is also atmospheric pressure p_0? It means that pressure of liquid in the very top layer is now not 0, but p_0. The increase in pressure between the top and the bottom happens in the same way and pressure at the bottom is

$$p(h) = p_0 + \rho g h.$$

As an example let us ask at what depth in a lake (or sea) pressure created by water is equal to normal atmospheric pressure, about 100 kPa. Density of water is about 1000 kg/m^3. We need to find h such that

$$\rho g h = p_0,$$

therefore

$$h = \frac{p_0}{\rho g} = \frac{100,000 \text{ Pa}}{1000 \text{ kg/m}^3 \cdot 10 \text{ N/kg}} = 10 \text{ m}.$$

So 10 meters of water create pressure about one atmospheric pressure. Similarly, 20 meters of water create twice the atmospheric pressure and so on. This pressure is very important to consider for scuba divers and submarines.

Another very important thing to understand about pressure in liquids is that it acts in the same way in all directions. Namely, force per unit area is the same at a given point, no matter how the area is oriented: vertically, horizontally or at an angle. Actually this allows us to understand that pressure is constant along any horizontal plane. If we consider an imaginary thin horizontal tube of liquid, it is in equilibrium, so horizontal forces acting on its ends must be balanced. If our tube has constant cross section area, this leads us to conclude that pressure at the endpoints is the same. So we conclude that pressure only depends on depth below the surface. Because of this our formula $p = p_0 + \rho g h$ actually works for arbitrary vessel shapes, not just ones with constant cross-section area.

Homework

1. Density of mercury is approximately 13,500 kg/m^3. What height of mercury column creates normal atmospheric pressure?

2. This problem will demonstrate why we did not bother with variation of pressure with height when discussing gas laws. Find pressure variation between the top and the bottom of a container with nitrogen at 27°C and average pressure 100000 kPa (one atmosphere). The container is 1 m high. How big is pressure variation compared to the average pressure? *Hint: you would need to know the gas density. Use definition of density and the ideal gas law. Do not forget how to relate number of moles to mass.*

***3.** Consider three vessels with water with different shape, as shown on the figure. They all are filled to the same level h and have the same cross section area at the bottom, but vessel A gets wider from bottom to the top and vessel C gets narrower. Clearly, the mass of water contained in them is not the same: there is more water in A than in B and more in B than in C. On the other hand, as we discussed, pressure at the bottom is the same for all of them and normal force acting on the water from the bottom is also the same (cross section area of the bottom is the same). So normal force on the bottom cannot cancel gravitational force for all of them. Resolve this apparent paradox.