

FIRST LAW OF THERMODYNAMICS.

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THEORY RECAP

Last time recap. Last time we discussed work done by gas and its' graphical representation. We have learned that in a process where pressure p of the gas is constant and its volume increases by ΔV , work A done by the gas is calculated as follows:

$$A = p\Delta V.$$

For cyclic processes total work done by the cycle is numerically equal to the area enclosed inside the cycle depicted on a $p - V$ diagram.

Quantity of heat. Imagine heating a gas by a burner, for instance. Burner supplies some energy to the gas. This energy coming from the burner is called quantity of heat and denoted by Q . Like all other kinds of energy, quantity of heat is measured in Joules.

First law of thermodynamics. Where does this energy go to? There are two possible ways. In order to see them, let's imagine first that while heating the gas we allow the piston to move in such a way that temperature of the gas stays constant. Temperature of the gas is the measure of its internal energy and as temperature does not change, internal energy stays constant. As volume increases, gas does some work because of the heat supplied. Energy is always conserved, and since energy does not accumulate in the system, work A is just equal to the supplied amount of heat in this process:

$$A = Q \text{ for } T = \text{const}$$

On the other hand, let us look what happens if we fix the position of a piston. Then gas does not do any work and clearly because of the burner its temperature increases. Back when discussing temperature we've learned that temperature is a measure of internal energy of a substance. When temperature increases, internal energy also grows. Let us denote change in internal energy by ΔE . Since volume is fixed, no work is done by the gas. Therefore due to energy conservation all of the supplied heat Q should be accumulated in the system in the form of internal energy:

$$\Delta E = Q \text{ for } V = \text{const}$$

What happens if we combine these two cases? Imagine we allow gas to change both its volume and temperature, which results in increase in internal energy of the gas by ΔE and work A done by the gas. The energy balance in this case works as follows: energy Q comes into the system from the outside, but energy A is spent on doing the work. Therefore the accumulation of internal energy is the difference between supplied and spent:

$$\Delta E = Q - A.$$

This is the first law of thermodynamics - essentially, nothing more but energy conservation law all over again. Another common form of writing it is as follows:

$$Q = \Delta E + A$$

with an interpretation that all the heat supplied to the system could go either into work done by the system or into change of internal energy.

Internal energy of the gas. For ideal gas there is a relation between internal energy and temperature, which is actually quite simple. In order to derive it, we will need two things. First, remember that temperature in Kelvins T is related to average kinetic energy of molecules of the substance as follows:

$$E_{kin\ av} = \frac{3}{2}kT$$

where $k = 1.38 \cdot 10^{-23}$ J/K is the Boltzmann constant. Second, remember that in an ideal gas there is no interaction between the molecules, so internal energy really just consists of kinetic energy. If we have n moles of gas, there are $N = nN_A$ molecules of the gas and the overall kinetic energy is

$$E = E_{kin\ av} \cdot N = \frac{3}{2}kTnN_A = \frac{3}{2}nRT.$$

where we used that $R = kN_A$, as we discussed some time ago. So internal energy of the gas has a very nice and simple expression in terms of temperature, number of moles and the universal gas constant.

Let me make a warning that there is a caveat in this derivation because of which it only works for very simple gas molecules which consist of only one atom (called monatomic gas). An example of such gas is helium, He . The caveat is that molecules which consist of several atoms could perform complicated motion - like rotation or oscillations, which we haven't accounted for in our derivation when looking at the kinetic energy. It is not very hard to account for, but it's beyond the scope of our course.

There is another common way to rewrite the formula for internal energy of a gas:

$$E = nC_V T,$$

where $C_V = \frac{3}{2}R$ for a monatomic gas is called molar heat capacity at constant volume. The reason for this name is that is volume is constant, as we discussed above, no work is done and

$$Q = \Delta E = nC_V \Delta T$$

So heat is proportional to amount of moles and change in temperature. The coefficient in such expression is normally called molar heat capacity, and our name additionally specifies that it is valid for constant volume processes.

HOMEWORK

1. 2 moles of helium are kept at constant volume and heated by a burner. How much does helium temperature increase if burner supplies heat 500 J?
2. While 5 moles of helium were heated by an alcohol burner, helium's temperature increased by 300° C and helium performed 8000 J of work. Every gram of burned alcohol releases 26,000 J of heat. Assume that due to heat losses only 30% of heat

released by the burner actually reaches the helium. What mass of alcohol was used to fuel the burner during this process?

- *3.** 1 mole of monatomic gas is heated at constant pressure. Find what quantity of heat should be supplied in order to increase temperature of the gas by 1°C . *Hint: use the expression for work done by gas and equation of state of ideal gas.*