

## WORK DONE BY GAS.

APRIL 18, 2021

### THEORY RECAP

**Last time recap.** Last time we discussed some ideal gas processes and their graphical representation. All of that discussion relied on ideal gas equation of state:

$$pV = nRT.$$

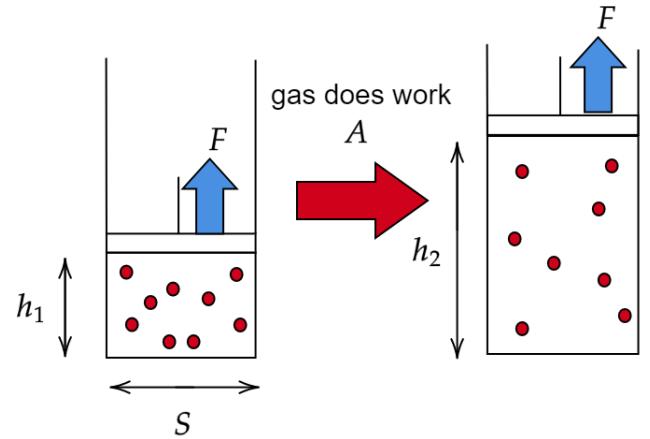
**Work done by gas.** Today we are considering an interesting connection between mechanics and thermodynamics. We will learn how heat could be transferred to work with an example of gas.

First, what is work? As you recall from mechanics, work is force times displacement. So in order to have work, we need to have force and displacement.

Consider our favorite setup: gas with pressure  $p$  in a container under a piston. If cross section area of the container (and the piston) is  $S$ , force acting on the piston could be found from definition of pressure:

$$p = \frac{F}{S} \implies F = pS.$$

Assume that because of the gas pressure piston moved height  $h = h_2 - h_1$  (see figure) above its initial position. Work done by the gas on the piston in this case is



$$A = Fh = pS(h_2 - h_1) = p(Sh_2 - Sh_1) = p(V_2 - V_1) = p\Delta V.$$

We used that product of cross section area and height is volume of the gas (final volume is denoted  $V_2$  and initial volume  $V_1$ ) and denoted change of volume by  $\Delta V$ . So, we have derived the formula for work done by gas: work is done whenever volume is changed and for processes at constant pressure it is equal to pressure times change in volume:

$$A = p\Delta V.$$

Remember that work could be positive or negative: it is positive when force and displacement point in the same direction and negative when they point in the opposite directions. Let us consider two possible cases to show that our formula for work of the gas actually gets this right. When volume changes, it either increases or decreases. When volume increases, force on the piston and displacement of the piston look in the same direction (up in our figure above). Therefore work should be positive - and from our formula  $A = p\Delta V > 0$  because  $\Delta V > 0$ . On the other hand when volume decreases, displacement is in the opposite direction (down on our figure) than the force (which still points up). Therefore work should be negative and from our formula  $A = p\Delta V < 0$  because  $\Delta V < 0$ .

There is a nice relation between work of gas and graphical representation of processes which we discussed previously.

Let us consider a cyclic process  $1 - 2 - 3 - 4 - 1$  consisting of two isobaric processes (1-2 and 3-4) and two isochoric processes (2-3 and 4-1), see figure to the right. We will ask what is the total work done by the gas during the cycle. Total work is of course equal to the sum of individual works done during every leg of the cycle:

$$A_{total} = A_{12} + A_{23} + A_{34} + A_{41}$$

Let us look at these terms individually. Start with  $A_{12}$ . The process 1-2 is isobaric, meaning that pressure is constant, so we could use our formula for work of the gas directly:

$$A_{12} = p_1(V_2 - V_1)$$

where notations are explained on the figure. We found that work in process  $1 - 2$  is numerically equal to area under the line 1-2 (which is just area of a rectangle), shown in red on the figure.

Now we need to find  $A_{23}$ . Process 2-3 is isochoric, which means volume does not change. Therefore piston does not move and no work is done:

$$A_{23} = 0.$$

Process 3-4 is again isobaric but volume is now decreased and therefore work turns out to be negative:

$$A_{12} = p_2(V_1 - V_2) < 0.$$

It is numerically equal to minus the area of the rectangle under line 3-4, shown in blue on the figure.

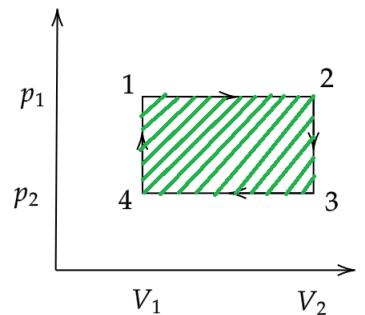
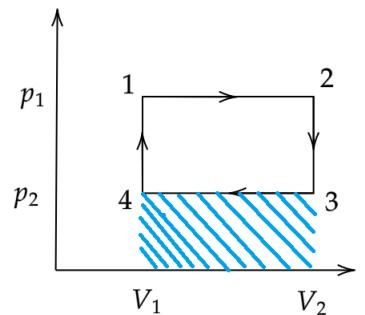
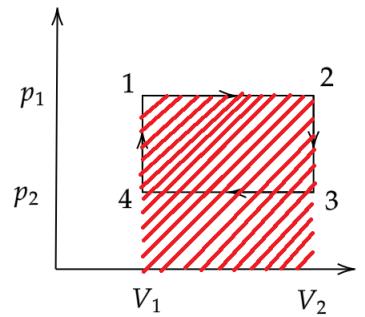
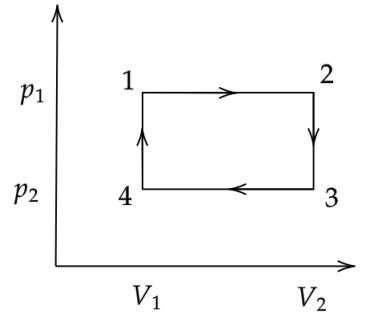
Finally, process 4-1 is again isochoric and no work is done, so

$$A_{41} = 0.$$

Summing up, we have

$$A_{total} = p_1(V_2 - V_1) + p_2(V_1 - V_2) = (p_1 - p_2)(V_2 - V_1)$$

We see that total work is numerically equal to the area of rectangle which represents our cycle in  $p - V$  coordinates, shown in green on the figure.



## HOMEWORK

1. There is a cylinder with a piston. The mass of the piston is 100 kg, its area is  $10 \text{ cm}^2$ . The cylinder contains 28 g of nitrogen at  $T_1 = 273 \text{ K}$ . The cylinder is heated up to  $T_2 = 373 \text{ K}$ . How does the piston position change? How did potential energy of the piston change? What work is done by the gas? Neglect atmospheric pressure.
2. How much hydrogen  $H_2$  (in grams) is in a cylinder with a piston if it performs work of 400 J being heated from 250 K to 680 K? The gas pressure was maintained constant.
- \*3. We did not have time to discuss this in class, but it is true for every cycle that work during the cycle is numerically equal to area of the region enclosed by the cycle. Using this fact, find total work done by the gas during the cycle 1-2-3-1 shown on the figure.

