IDEAL GAS PROCESSES. GRAPHICAL REPRESENTATION.

APRIL 11, 2021

Theory recap

Last time recap. Last time we discussed ideal gas equation of state:

$$pV = nRT.$$

We also learned how to find the number of moles knowing mass m and molar mass M:

$$n = \frac{m}{M}$$

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Processes with ideal gas. Our ultimate goal is to understand how gases could be used in machines to extract work from heat. For that we need to learn a bit about processes which could happen to gases and how to describe them conveniently. We have already understood that gas in a given state is characterized by its pressure, volume and temperature and amount of moles. Assuming that we fix amount of moles and don't change it, we only need to know two parameters, for example pressure and volume, to specify the state of the gas. Temperature then could be found using the ideal gas equation of state.

Graphically we could represent state of the gas as a point in $(p - V)$ coordinate plane: every point corresponds to some particular values of pressure and volume. For example, let us take one mole of some gas with pressure $p_0 = 101,339$ Pa and volume $V_0 = 0.0224$ m^3. This state is represented as a point on figure 1.

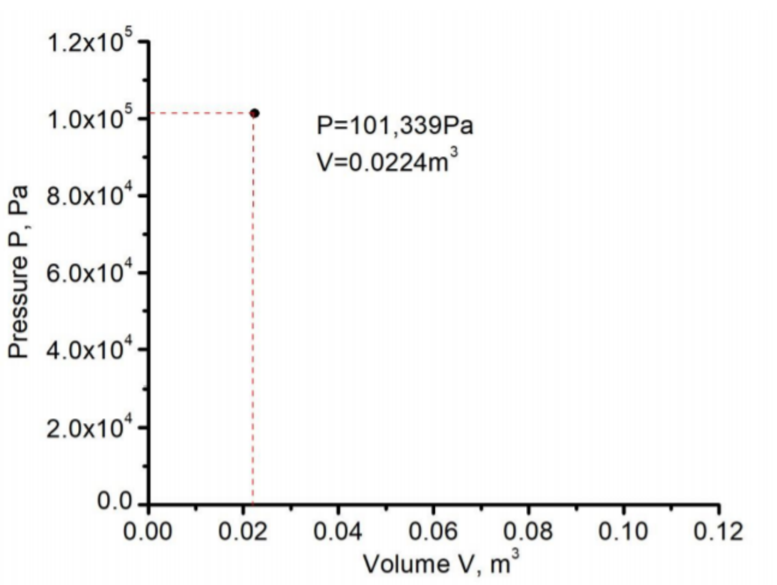

Figure 1. The point in $p - V$ coordinates represents state in which gas has pressure 101,339 Pa and volume 0.0224 m^3.

Knowing pressure and volume we could find temperature in this state:

$$p_0 V_0 = nRT_0 \implies T_0 = \frac{p_0 V_0}{nR} = 273.16\text{K}$$

Now let us decrease pressure of the gas while keeping the volume constant (processes at constant volume are called isochoric). In this process the gas will go through many consequent states, all with the same volume. On our plot it will be represented by a continuous line with every point on it corresponding to some intermediate state. Constant volume means the volume coordinate is fixed, so this should be a vertical line. Its' endpoint will be at the final pressure which we will take to be $p_1 = 20,000$ Pa.

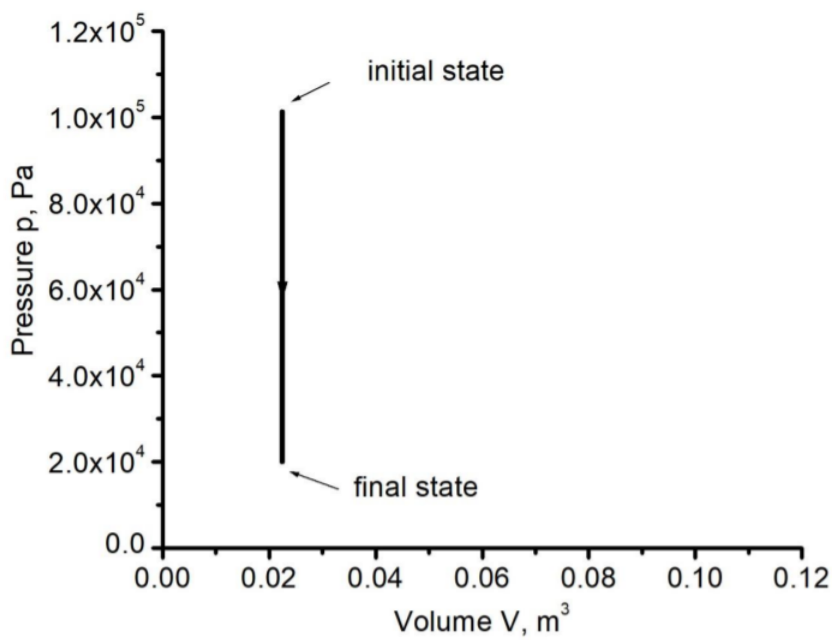

FIGURE 2. Vertical line in $p - V$ coordinates represents a process at constant volume, also called an isochoric process.

We could find the temperature T_1 in the final state with pressure p_1 by using either equation of state of ideal gas as above, or Gay-Lussac's law, which gives us

$$\frac{p_1}{T_1} = \frac{p_0}{T_0} \implies T_1 = T_0 \frac{p_1}{p_0} = 54 \text{ K}.$$

There is a possible caveat here, that 54 K is actually a really low temperature at which many gases, for example nitrogen or oxygen become liquid and therefore can not be described by ideal gas equation of state. But there are gases which only condense at much lower temperature, such as helium (at 4.2 K). So let us assume that our gas is helium and it still can be well approximated by ideal gas at 54 K.

Now let us continue with our process. The next part will be done at constant pressure (processes at constant pressure are called isobaric) and increasing volume. Let us take the final volume to be $V_2 = 0.113 \text{ m}^3$. Process at constant pressure is represented by horizontal line on our plot:

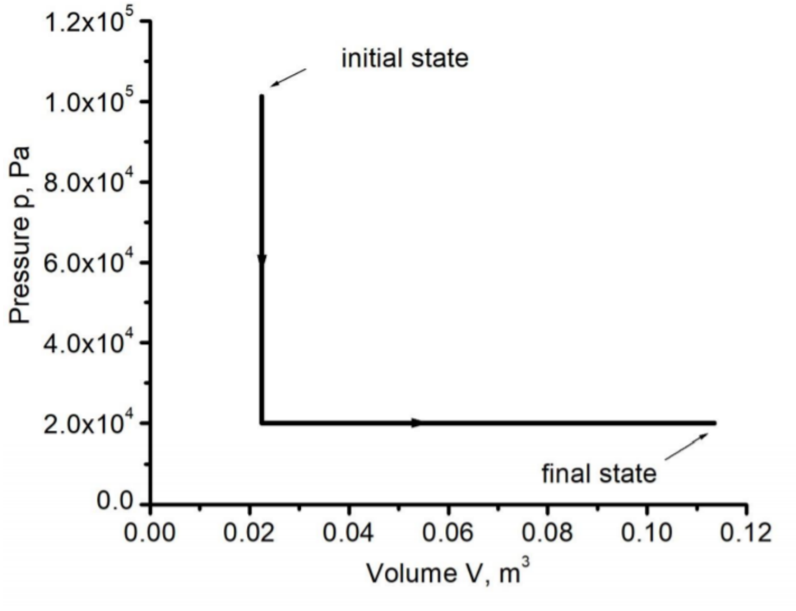

FIGURE 3. Horizontal line in $p-V$ coordinates represents a process at constant pressure, also called an isobaric process.

We could calculate temperature in the new final state this time using Charle's law:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \implies T_2 = T_1\frac{V_2}{V_1} = 273.16 \text{ K}.$$

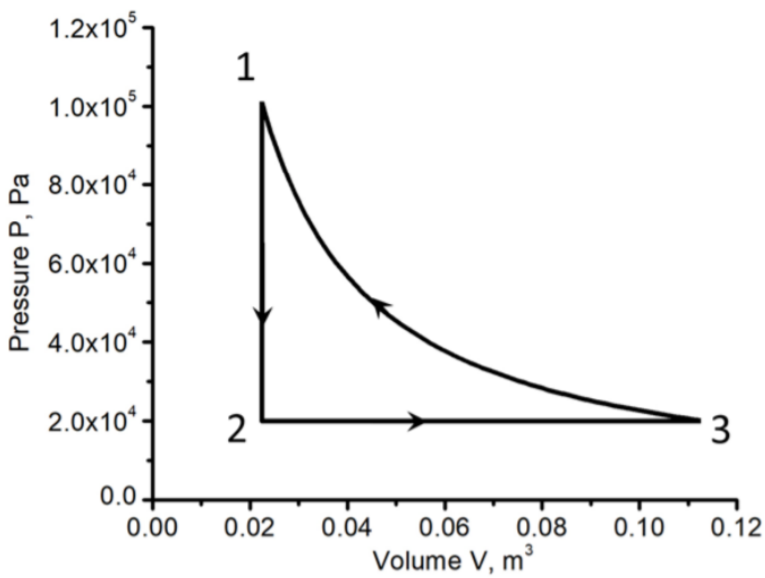

FIGURE 4. Hyperbola 3-1 in $p-V$ coordinates represents a process at constant temperature, also called an isothermal process.

So we have reached the same temperature as we had initially. We would like to return to the initial state because in order for us to be able to repeat the process again and again it should be cyclic. The simplest opportunity now is to compress the gas at constant temperature, or isothermally. As we discussed some time ago, on the $p-V$ plot the corresponding curve is hyperbola, as shown on figure 4. The equation of this curve could be found from equation of state of ideal gas:

$$pV = nRT \implies p = \frac{nRT}{V} = \frac{2270\ \text{J}}{V}.$$

Homework

1. Draw the cyclic process shown in Figure 4 in the coordinates p, T and V, T.

2. Consider the following cyclic process performed with 3 moles of ideal gas. We start from pressure 10 kPa and volume 2 m^3 (point 1). Then we isobarically compress the gas until volume reaches 0.5 m^3 (point 2). Then at constant volume pressure is increased up to 40 kPa (point 3). After that keeping the pressure constant we bring the volume up to the initial value 2 m^3 (point 4). Finally pressure is isochorically reduced and the gas comes back to point 1. Draw this process in $p-V$ coordinates and find temperature of the gas in points 1,2,3 and 4.

***3.** There is a cyclic process shown in a Figure 5 below. Show on the graph the points corresponding to the gas states with highest and lowest temperature.

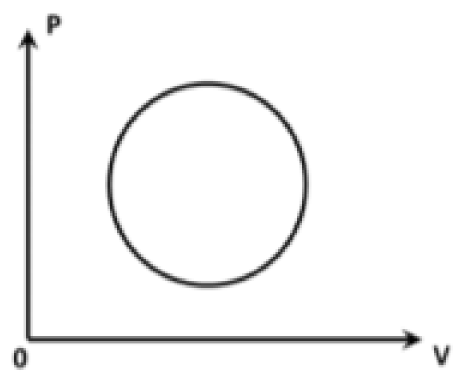

Figure 5. To problem *3