EQUATION OF STATE OF IDEAL GAS.

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Theory recap

Last time recap. Last time we discussed combined gas law:

$$\frac{pV}{T} = const.$$

A question which we started exploring is what this constant is equal to. We discussed that it has to be proportional to the number of molecules (or atoms) in the gas. Since this number is very large for even small portions of normal gases, it is convenient to use moles for calculating the number of molecules. As we discussed, 1 mole contains $N_A = 6 \cdot 10^{23}$ molecules. In other words, we define the amount of substance as

$$n = \frac{N}{N_A}.$$

Equation of state of ideal gas. Summing up, combined gas law is:

$$\frac{pV}{T} = const \propto n$$

And this is basically it! We have established all the physical content of this formula and now we just need to say what the proportionality coefficient is! Lo and behold, the ideal gas equation of state is:

$$pV = nRT$$

Everything in this formula is already familiar to us except R. R is called gas constant (or universal gas constant) and it is equal to

$$R = 8.31\ \frac{\mathrm{J}}{\mathrm{mole} \cdot \mathrm{K}}$$

Equation of state of ideal gas contains combined gas law and through it the three individual gas laws that we discussed. To see it, we consider a fixed amount of gas: $n = const$. Then we see

$$\frac{pV}{T} = nR = const.$$

This is combined gas law. But equation of state of ideal gas teaches us more than that. It tells us that all gases, no matter what atoms or molecules they consist of, have the same relation between four basic parameters: pressure, volume, temperature and amount of substance. So if you take the same amount of molecules of hydrogen or water vapor or chlorine, you wound not be able to distinguish them by the pressure they produce at given volume and temperature. If they have the same volume and temperature, they will have the same pressure. This is the great universality which is only found among gases because of their physical simplicity: in an ideal gas approximation they are just a bunch of molecules flying around without interacting with each other.

As an example of this universality let us calculate the volume that 1 mole of any gas would take at normal conditions: normal atmospheric pressure $p_0 = 101.3$ kPa and temperature $T = 0^\circ$ C $= 273$ K. From the equation of state of ideal gas we get

$$V = \frac{nRT}{p_0} = \frac{1 \cdot 8.31 \cdot 273}{101,300} \text{m}^3 = 0.0224 \text{ m}^3 = 22.4 \text{ L}$$

The famous answer is 22.4 liters.

Equation of state of ideal gas even works for mixture of gases: if we take 2 moles of oxygen and 3 moles of nitrogen, we will have equation of state for 5 moles of gas.

As a little side note, and for now just a fun fact, you can recall that in our course we have already had two physical constants with the number 23 in power of 10. One is Avogadro number $N_A = 6.02 \cdot 10^{23}$ mole^{-1} and the other is the Boltzmann constant which we discovered some time ago talking about physical meaning of temperature: $k_B = 1.38 \cdot 10^{-23}$ J/K. If we multiply these two numbers, powers of 10 mutually cancel and we get

$$N_A k_B = 6.02 \cdot 1.38 \frac{\text{J}}{\text{mole} \cdot \text{K}} = 8.31 \frac{\text{J}}{\text{mole} \cdot \text{K}} = R$$

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In other words, we get exactly the universal gas constant! Number and dimension match perfectly. So this constant is related to the Avogadro number and Boltzmann constant. In your future studies of physics you will surely learn why, but it is a bit beyond the scope of our current course.

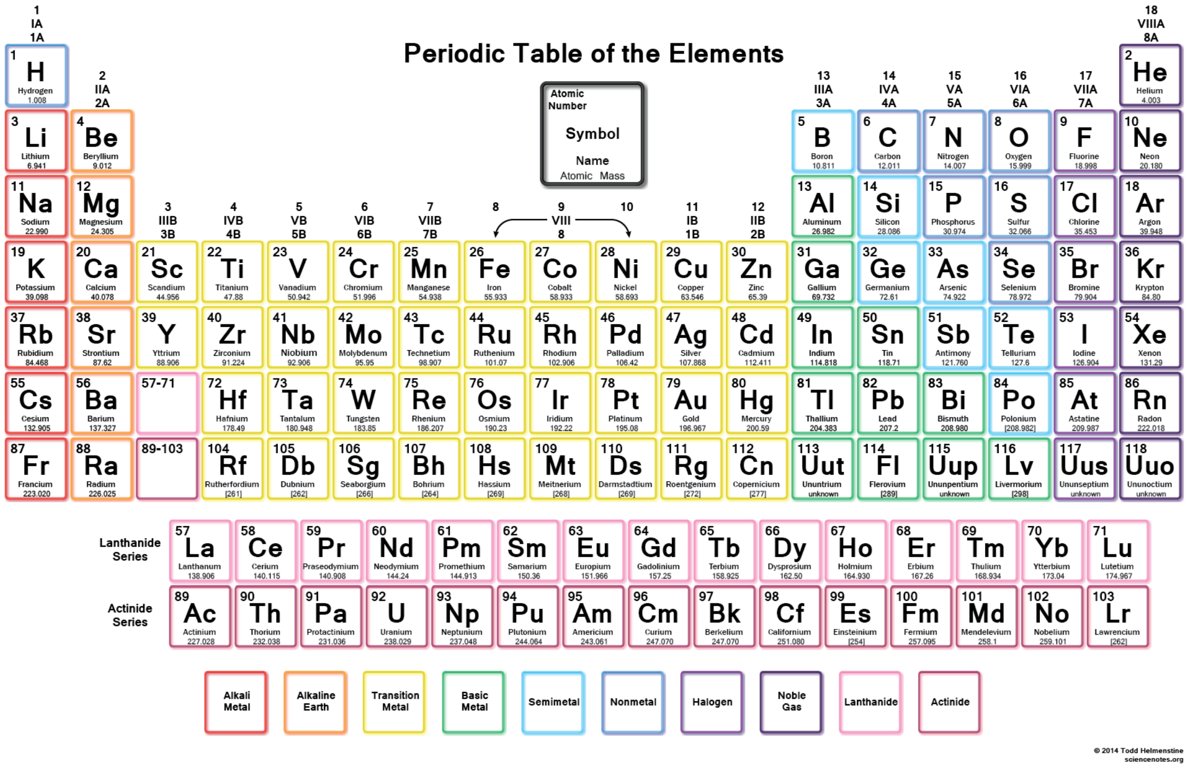
Molar mass. But how do we know the amount of substance of some matter? We could not count the number of molecules, of course. What we can easily do is measure the mass of substance. Could we maybe find number of moles from the mass? Yes! Each molecule of a substance has a particular mass, so $6 \cdot 10^{23}$ of these molecules also have a particular mass which is called molar mass and denoted by M. Now, if we have m kilograms of substance with mass M, the amount of substance is

$$n = \frac{m}{M}.$$

Equation of state of ideal gas could be written in terms of mass and molar mass:

$$pV = \frac{m}{M} RT$$

Molar mass from periodic table. How could we find molar mass of some substance? As we have seen, molar mass is related to mass of molecules of the substance. Mass of a molecule is equal to sum of masses of atoms comprising this molecule. And masses of atoms could be found in the periodic table of elements, which contains a lot of useful information about all the atoms.



There is a simple algorithm of finding molar mass from periodic table. First of all, we need to locate atomic mass in the periodic table: it is the lowest number in each cell. For example, for the first element - hydrogen (H) we can see the atomic mass is 1.008 which could be rounded to 1. This is exactly the molar mass of a hydrogen atom, measured in gram/mole. So, if we take 1 mole of hydrogen atoms, or $6 \cdot 10^{23}$ hydrogen atoms, their mass will be $M(\mathrm{H}) = 1$ g/mole. If we take 1 mole of carbon atoms, their mass will be $M(\mathrm{C}) = 12$ g/mole (find carbon C in the table above and verify that its' atomic mass is about 12).

Now, if we talk about molecules, molar mass is sum of molar masses of atoms building the molecules. For instance, hydrogen molecule is H_2 which means it consists of two hydrogen atoms. Therefore, one mole of hydrogen molecules has mass equal to two molar masses of hydrogen atoms:

$$M(\mathrm{H_2}) = 2 \cdot M(\mathrm{H}) = 2 \text{ g/mole}.$$

Let us do one more example. Consider a water molecule H_2O which consists of two hydrogen atoms and an oxygen atom. We already know that molar mass of hydrogen atom is 1 g/mole, but we need to know molar mass of oxygen atom. We look at the periodic table and find that oxygen O has atomic mass around 16. This means that molar mass of oxygen atom is $M(\mathrm{O}) = 16$ g/mole. So we find molar mass of water:

$$M(\mathrm{H_2O}) = 2 \cdot M(\mathrm{H}) + M(\mathrm{O}) = 2 + 16 = 18 \text{ g/mole}.$$

Homework

1. What is the volume of 3 moles of an ideal gas at the temperature of 57°C and pressure 150 kPa?

2. There is a 1 liter bottle filled with water at 27°C. The water is liquid at this temperature because there is attracting force between the molecules. Imagine that we have suddenly "turned off" this attracting force. What is the pressure in the bottle now? *Hint: mass of 1 liter of water is 1 kg.*

3. Find molar mass of carbon dioxide CO_2 using periodic table. Using it, find mass of carbon dioxide in a 10 liter cylinder if it has temperature T=13°C and pressure $P = 9 \cdot 10^6$ Pa.

***4.** Gas is in a vessel at a pressure 2 MPa and temperature 27° C. After its' temperature is increased by 50° C, half (by mass) of the gas escapes the vessel. What is pressure in the new equilibrium state?