GAS LAWS CONTINUED

MARCH 14, 2021

Theory recap

Boyle's law recap. Last time we discussed Boyle's law. We learned that at constant temperature increasing pressure of a gas leads to decreasing volume, according to the formula:

$$\frac{p_2}{p_1} = \frac{V_1}{V_2}.$$

An equivalent way of saying it is that the product of pressure and volume is constant when temperature is constant:

$$pV = const \quad \text{for} \quad T = const$$

There is an intuitive explanation of why Boyle's law is valid. As we discussed, pressure is produced by gas atoms or molecules colliding with the walls of the container. There are lots and lots of the collisions every second and on average they lead to some particular pressure. Imagine that we increase the volume of the gas. Then is becomes more rarefied and in the same time period less collisions of gas particles with the walls happen. If there are less collisions, the average force is smaller and therefore pressure is smaller. This is basically how Boyle's law works.

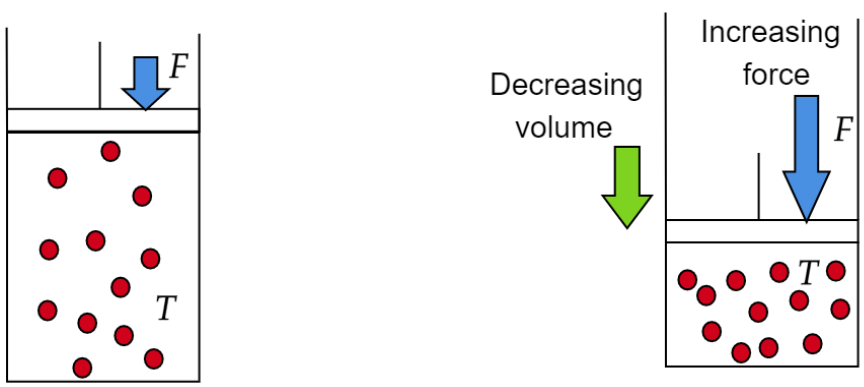

Figure 1. **Left figure:** Initially gas has some volume. Molecules collide with the piston and produce some force. An equal force is needed to balance it. **Right figure:** Volume of gas is decreased. Molecules are packed more densely and collide with the piston more often leading to the larger force. So, pressure of the gas increases.

Charle's law. Today we will further learn what happens if we change temperature. As we discussed, there are two other gas laws in each of them one of the two remaining quantities (pressure or volume) is kept fixed. Let us begin with fixed pressure.

We could keep the same pressure by applying a constant force on the piston. We can even not apply any force ourselves - there will still be atmospheric pressure. In addition gravity force acting on the piston will also produce some pressure. Then we could heat the gas and measure how does it affect the gas volume. This way we obtain Charle's law that says that volume is proportional to the temperature measured in Kelvins:

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}.$$

An equivalent way of writing Charle's law is:

$$\frac{V}{T} = const \quad \text{for} \quad p = const.$$

For example, if at 300 K some gas had volume 300 cm^3 then at 400 K and at the same pressure it will have volume 400 cm^3, at 500 K volume will become 500 cm^3 etc. Let us try to visualize this law on a plot. This time our axes will be temperature T versus volume V. T proportional to V corresponds to a straight line on this plot. Note that this line goes through the point $(0, 0)$ so it predicts that at 0 K the gas will have zero volume. In reality, this is nonsense - nothing could have zero volume. The truth is, Charle's law and other gas laws are established for the ideal gases which are only an approximation to real gases. In an ideal gas there is no interaction between its' molecules at all. In real gases there is always some interaction but if the gas is rarefied enough it could be neglected. So ideal gas is a good approximation for a gas of small density. However as gas becomes denser approximation by an ideal gas becomes worse. So for very small volumes it is not a good approximation anymore and Charle's law (and other laws) do not hold anymore.

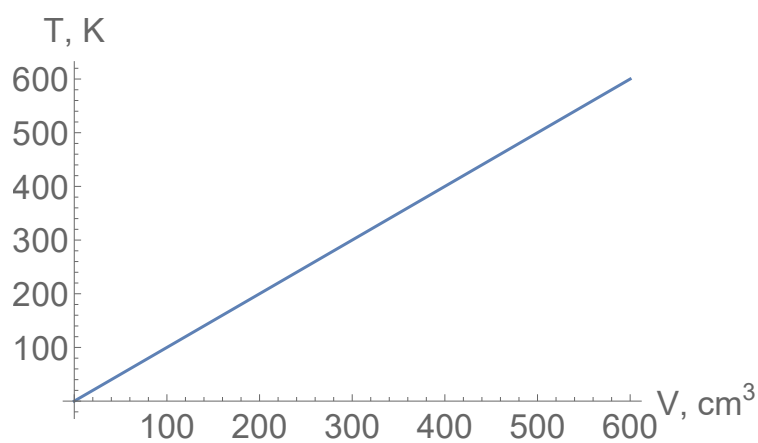

FIGURE 2. The line of constant pressure with temperature in Kelvins

Why is it important that temperature in Charles law is measured in Kelvins? Imagine that instead of Kelvins we measured temperature in Celsius. As we know,

$$T = t - 273$$

where T is temperature in Kelvins and t is temperature in Celsius. So if we want to express temperature in Celsius in Charle's law we would get

$$\frac{V_2}{V_1} = \frac{t_2 - 273}{t_1 - 273}$$

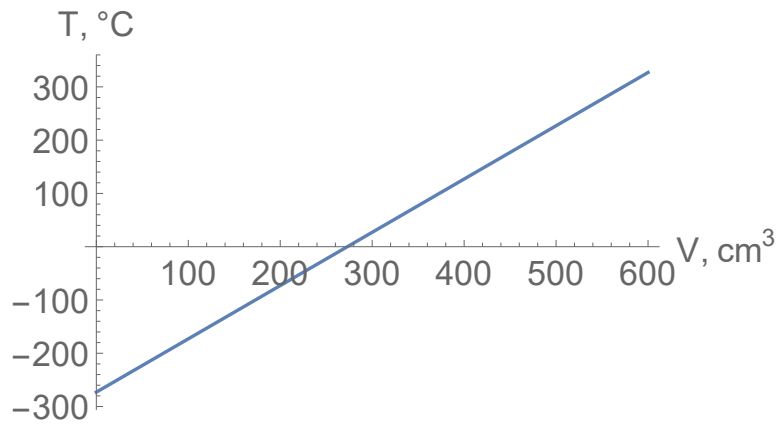

FIGURE 3. The line of constant pressure with temperature in degrees Celsius

Does not look so nice, does it? If we look back at our plot and express temperature in Celsius, the straight line would not go through the point $(0, 0)$ anymore. Instead, zero volume would correspond to temperature -273 °C. Historically, this was how the scientists first noticed the existence of an absolute zero of temperature. To sum up, we see that Kelvin scale allows us to write Charle's law in a much simpler form.

How about a microscopic reason behind Charle's law? Remember that temperature is a measure of average kinetic energy of the molecules. If we increase temperature, molecules have more kinetic energy. As a result, each individual collision of a molecule with the wall produces a larger force. But if the volume becomes bigger and collisions are more rare, there are less collisions. For less collisions of increased force we get the same average force - or the same pressure. So if temperature grows together with volume, the pressure is constant. This is precisely Charle's law.

Gay-Lussac's law. Having discussed Charle's law we are ready for the third gas law. Let us now keep volume of the gas fixed and measure how pressure changes when we change temperature. This way we discover Gay-Lussac's law: pressure is proportional to the temperature, once again measured in Kelvins:

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}.$$

An alternative way of writing Gay-Lussac's law is:

$$\frac{p}{T} = const \quad \text{for} \quad V = const.$$

It looks similar to Charle's law except that volume and pressure are interchanged. Once again, it is crucial that temperature is measured in Kelvins.

Homework

1. A cylinder is filled with gas. The pressure inside is 10000 Pa, the temperature is 20°C. We increase the temperature to 100°C. What happens to the pressure inside the cylinder? Calculate the new pressure.

2. Gas was heated from 27°C to 39°C but its pressure was maintained the same. By what percentage did volume of the gas increase?

***3.** Pressure of air in a bottle at 7°C is equal to the atmospheric pressure 100 kPa. How much does one need to heat the bottle so that a cork closing the bottle will be pushed out? Without heating the cork could be pulled out by force 10 N. Cross-section area of the cork is 2 cm^2.