THERMAL ENERGY

FEBRUARY 7, 2021

Theory recap

Internal energy. Last time we discussed temperature. We learned that higher temperature corresponds to larger internal kinetic energy. Actually, internal kinetic energy is not the only kind of internal energy - there is also internal potential energy. Together, internal kinetic and potential energy comprise internal energy.

Internal kinetic energy grows with temperature and we know exactly how. Internal potential energy also grows with temperature but how exactly it grows depends on the structure of an object. Anyway, if temperature of an object increases, its internal energy grows. If temperature grows more, internal energy grows more as well. Change in internal energy is proportional to the change of temperature:

$$\Delta E_{int} = C\Delta t$$

where ΔE_{int} is the change in internal energy, Δt is change in temperature and C is the coefficient known as heat capacity. The larger heat capacity is, the bigger is change in internal energy for the same change in temperature.

Specific heat capacity What does heat capacity of an object depend upon? Consider a brick. Let us change its temperature from some t_1 to some t_2 so that its internal energy changes by $\Delta E_{brick} = C(t_2 - t_1)$, where C is heat capacity of the brick. By how much does internal energy of a half of the brick change under the same temperature change? It is clear that internal energy should be split equally between the two halves. Therefore, $\Delta E_{half} = \dfrac{1}{2}\Delta E_{brick} = \dfrac{1}{2}C(t_2 - t_1)$. We see that heat capacity of the half of the brick is $C_{half} = \dfrac{1}{2}C$.

So, if the object is built out of smaller parts, the object's heat capacity is the sum of heat capacities of these parts. It should not be surprising - this is how energy works, including internal energy. In other words, heat capacity is proportional to mass of the object. Heat capacity of 1 kilogram of some material is known as specific heat capacity of this material. Specific heat capacity is denoted by c, so we have a formula:

$$C = cm$$

for heat capacity of an object of mass m made out of material with specific heat capacity c. This allows us to rewrite the formula for change of internal energy as

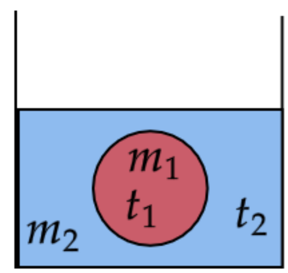
$$\Delta E_{int} = cm\Delta t.$$

Specific heat capacity is a property of the material which is measured experimentally. Units of specific heat capacity are J/(kg·°C). For example, for water $c = 4200$ J/(kg·°C). For iron it is $c = 460$ J/(kg·°C).

Heat transfer Having talked about internal energy, let us now talk about how it could be changed. In other words, how do we make an object hotter? A common way is heat transfer from another hot object. We know very well that if two objects of different temperature are brought into contact their temperature tends to become equal as time passes. For example if you put an apple in the refrigerator soon the apple will become cold because the apple is in contact with cold air and surface in the refrigerator. The fact that temperatures of objects in contact become equal is referred to as reaching thermal equilibrium. Our understanding of temperature would allow us to understand the basic underlying mechanism of reaching thermal equilibrium. Remember that temperature is a measure of internal kinetic energy. So if one object has higher temperature than the other, it means that kinetic energy of atoms and molecules in the first object is larger than in the second object. When these objects are brought into contact, their atoms and molecules collide with each other and during these collisions the ones with higher kinetic energy give their extra energy to the ones with lower kinetic energy. As a result, kinetic energies of atoms and molecules in both objects tend to become the same. Therefore, temperatures tend to become the same.

The situation when internal energy is transferred from one object to another is known as heat transfer. In practice we could use the formulas for internal energy change in order to solve various problems. In particular, a common task is to find the equilibrium temperature after two objects of particular temperatures are brought into contact.

Let us look at such an example: we take a piece of iron of mass $m_1 = 100$ g which has temperature $t_1 = 100°$ C and put it in a container with $m_2 = 200$ g of water at temperature $t_2 = 20°$ C. What temperature will water and iron reach after coming to thermal equilibrium? As often happens in such problems, we will assume that the container with water is well insulated from the environment, so no heat could flow between our system of water + iron and the environment.



Let us call the final temperature of water and iron t_3 - that is what we need to find. We would also need specific heat capacities of iron and water which we will call c_1 and c_2 correspondingly (their numerical values were provided above). The most important principle we should use is energy conservation law applied to internal energy. Since the system is insulated and no mechanical energy is dissipated and no work is being done, total internal energy must stay the same. This means that the change in total internal energy is 0:

$$\Delta E_{tot\ int} = \Delta E_1 + \Delta E_2 = 0$$

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Here ΔE_1 is change of internal energy of iron and ΔE_2 is the change of internal energy of water. Using our general formulas we get:

$$\Delta E_1 = c_1 m_1 \Delta t_1 = c_1 m_1 (t_3 - t_1);\ \Delta E_2 = c_2 m_2 \Delta t_2 = c_2 m_2 (t_3 - t_2)$$

where $\Delta t_1, \Delta t_2$ are changes of temperature of iron and water correspondingly and are found as final temperature minus initial temperature. Plugging this back to our energy conservation condition we get

$$c_1 m_1 (t_3 - t_1) + c_2 m_2 (t_3 - t_2) = 0$$

Now the physics part of the problem has ended and we just need to mathematically solve the equation for t_3. To do it we gather all the terms with the unknown t_3 on the left hand side of the equation and all the terms without t_3 on the right hand side:

$$c_1 m_1 t_3 + c_2 m_2 t_3 = c_1 m_1 t_1 + c_2 m_2 t_2 \Longrightarrow t_3 = \frac{c_1 m_1 t_1 + c_2 m_2 t_2}{c_1 m_1 + c_2 m_2}$$

.

Now we just need to plug in the numbers (remembering that 100 g = 0.1 kg):

$$t_3 = \frac{460 \cdot 0.1 \cdot 100 + 4200 \cdot 0.2 \cdot 20^\circ}{460 \cdot 0.1 + 4200 \cdot 0.2} C = 24^\circ C$$

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We see that the answer is 24° C, very close to the initial temperature of the water. So water temperature changed just a bit while iron temperature changed a lot. This is so because in our example heat capacity of water ($4200 \cdot 0.2 = 840$ J/°C) is much bigger than heat capacity of iron ($460 \cdot 0.1 = 46$ J/°C) so the same (in absolute value) change in internal energy results in a drastically different change of temperature.

Homework

1. Mr. X does not like when his morning coffee is too hot so he adds some cold milk to it. Initially the coffee is at boiling temperature (100°C) and milk is just out of the fridge (10 ° C). How much milk does Mr. X have to add to 150 g of coffee in order for the mixture to have temperature 65° C? You may assume that both coffee and milk have the same specific heat capacity as water.

2. A 500 gram cube of lead is heated from 20 ° C to 80 ° C. How much energy was required to heat the lead? The specific heat capacity of lead is 160 J/kg·°C.

***3.** Some object with initial temperature $t_1 = 100^\circ$ C is put in a glass with water with initial temperature $t_2 = 10^\circ$ C. After some time thermal equilibrium established and temperature became $t = 40^\circ$. Then another object, completely the same as the first one and also with initial temperature $t_1 = 100^\circ$ C was put in the same glass with the first object still in it. What will the resulting temperature t' be?