TEMPERATURE

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Theory recap

Introduction. Last time we discussed the relation between work and energy. That concluded our discussion of mechanical energy. We learned that there is kinetic energy and potential energy. We learned that mechanical energy of some object could be changed if some force performs work.

Mechanical energy is not the only form of energy. For example, if some objects slides on a surface with friction, kinetic energy of this object decreases. But no other object gains speed or moves up, so mechanical energy is just decreasing. But energy is never lost completely, it is just transferred to other forms - like in this case to thermal energy. Today we start discussing thermal energy and thermal phenomena. We begin this discussion with a very familiar but also a very interesting concept - temperature.

What is temperature. Every one of us can tell hot from cold by how it feels like, but what does it actually mean that something is hot or cold? Where does this notion originate physically? The fact that temperature can be increased by friction tells us that it has something to do with energy, but what is this energy?

Hopefully you are already familiar with the fact that all objects around us are built out of very small constituents: atoms and molecules. These tiny bits of matter are constantly moving at very high speeds, even if the object as a whole is motionless. They collide with each other and change the direction of their motion very often, so the object as a whole stays at rest. We will discuss this a little further when we talk about phases of matter in several weeks.

But since atoms and molecules constantly move, they have kinetic energy, right? Kinetic energy is present for any object with mass and speed, no matter how tiny it is. Atoms and molecules have mass, even though it is small, and speed, which is in fact not so small, as we are going to see. This kind of kinetic energy that is associated with internal motion of constituents of an object and not with the motion of an object as a whole is called internal energy.

Temperature is the measure of internal energy! Basically, the higher temperature is, the faster atoms and molecules move inside an object (so their kinetic energies are higher and so internal energy is bigger). This was actually one of big achievements of physics to discover that temperature that everyone feels is really connected to constant motion of tiny particles, which comprise everything around us and are invisible to the naked eye.

Temperature scales. Let us discuss different temperature scales. In order to define a scale we need to set up two reference points and assign two different temperatures to them. A convenient choice is to use freezing point of fresh water as one reference point and boiling (at normal atmospheric pressure) point of water as the other reference point. These reference points are particularly easy to reproduce (because almost everywhere and at any time one has access to fresh water) which is important for a widespread use. For precision scientific goals they might not be the best, but we will not discuss it further.

For example, in Fahrenheit scale freezing point of fresh water corresponds to 32° F and boiling point corresponds to 212° F. 0° F corresponds to freezing temperature of a particular solution of water and salt. 100° F is slightly above the normal temperature of a human body.

Another scale which is common in countries with metric units is Celsius scale. In Celsius scale freezing point of water is assigned $0°$ C and boiling point of water - $100°$ C so it is really built around these water reference points.

If one wants to go between these scales, a conversion formula exists. If t_F is temperature in Fahrenheit and t_C is the corresponding temperature in Celsius, they are related as follows:

$$t_F = 32 + \frac{9}{5} t_C$$

Let us check that reference points are obtained correctly: for $0°$ C (freezing point of water) this formula gives $32 + 0 = 32°$ F which is correct. For $100°$ C (boiling point of water) this formula gives $32 + \frac{9}{5} \cdot 100 = 32 + 180 = 212°$ which is also correct. So this formula could be used to find Fahrenheit temperature corresponding to a Celsius temperature. For example, if you go to Spain and see that sea water temperature is $20°$ C you could do this calculation and find that it is $68°$ F. Then you will be able to judge how comfortable it is.

Both Fahrenheit and Celsius scales are quite convenient for everyday purposes but there is something that they lack related to the physical meaning of temperature. We learned that temperature corresponds to the internal kinetic energy of atoms and molecules. Kinetic energy could never be negative (recall $\frac{mv^2}{2}$ - mass m is always positive and square of any number is larger or equal to zero, so kinetic energy can not be negative). But Celsius and Fahrenheit temperatures could be negative. How is it possible? It is because choice of zero in both Celsius and Fahrenheit does not correspond to zero of kinetic energy. It makes sense to define a temperature scale with zero exactly corresponding to zero internal kinetic energy - the so-called absolute zero of temperature. Kelvin scale is defined this way: it begins from absolute zero and has the same increment as the Celsius scale. So Kelvin scale does not have negative temperatures: the absolute zero is at 0 Kelvins. In Celsius scale absolute zero is approximately at $-273°$ C. Conversely, the zero of Celsius scale (water freezing point) is 273 K (Kelvins are used without a $°$ sign). And $100°$ C (water boiling point) is 373 K. The general relation between Celsius and Kelvin scales is therefore

$$T = t_C + 273$$

where T is temperature in Kelvins and t_C is the corresponding temperature in Celsius.

Relating internal kinetic energy and temperature. Kelvin scale is especially convenient for relating temperature and average internal kinetic energy of atoms and molecules. This relation is as follows:

$$E_{kin} = \frac{3}{2} kT$$

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where T is temperature in Kelvins and $k = 1.38 \cdot 10^{-23}$ J/K is called the Boltzmann constant. Boltzmann constant is the coefficient between average internal kinetic energy of atoms and molecules and temperature. It is a very small number, so kinetic energies of atoms and molecules are very small. But this is expected, because their mass is very tiny. Now we could estimate what velocities do atoms and molecules have at usual temperatures.

As an example let us take an oxygen molecule O_2, which has mass $m_{O_2} = 2.7 \cdot 10^{-26}$ kg. What is the average speed of such a molecule at temperature 300 K? (this temperature is close to usual room temperature and is often used in estimates for convenience, because it is a round number)

$$\frac{m_{O_2}v^2}{2} = \frac{3}{2}kT \Longrightarrow v^2 = \frac{3kT}{m_{O_2}} \Longrightarrow v = \sqrt{\frac{3kT}{m_{O_2}}} = 680\ m/s$$

Compared to a speed of commercial airplane, about 280 m/s we see that oxygen molecules at room temperature move about two and a half times faster! But normally they don't travel far because due to collisions they change direction of motion very often.

Thermal expansion. Finally, let us discuss one more phenomenon related to temperature: thermal expansion. As we learned, at higher temperature atoms and molecules move with higher speeds. Because of this they tend to get farther from each other and therefore size of the object increases slightly. How much it increases depends on the initial size, change of temperature and material. For example, a thin rod of length l has change of length Δl due to temperature change ΔT as follows:

$$\Delta l = \alpha l \Delta T$$

So change of length is proportional to the initial length, change of temperature (final temperature minus initial) and a coefficient α which is called linear thermal expansion coefficient and which is characteristic of a given material.

If our object is characterized by volume V, like liquid or gas, its volume also changes due to thermal expansion according to a similar formula:

$$\Delta V = \beta V \Delta T$$

where β is called volumetric thermal expansion coefficient.

Homework

1. Derive a general formula for calculating temperature in Celsius from temperature in Fahrenheit. Use it to calculate 80° F in Celsius.

2. What is the temperature of a human body in the Kelvin scale?

3. Temperature in the room is increased from 15° C to 39° C. How many times did the average kinetic energy of the air molecules increase?

***4.** Old-fashioned thermometers used mercury for measuring temperature. Their work is based on thermal expansion. Assume that a thermometer contains $V = 0.1$ cm^3 of mercury most of which is located in a big reservoir at the bottom. The remaining mercury goes up a tube with radius $r = 0.03$ cm. Volumetric thermal expansion coefficient for mercury is $1.8 \cdot 10^{-4}\ \frac{1}{°\mathrm{C}}$. Find the distance between two neighboring marks with temperature difference 1 ° C between them (for example, 36° C and 37° C marks).