WORK AND ENERGY

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Theory recap

Last time we discussed potential energy. We learned that total mechanical energy, which is the sum of kinetic and potential energies, is conserved in some cases - like when the only forces gravity and normal force. Today we will further discuss how presence of other forces alters the law of conservation of mechanical energy.

As a reminder, previously we discussed another conserved quantity - momentum. Momentum changes when there is an external force. Momentum change is equal to impulse - a product of force and time.

Forces could change mechanical energy as well. For energy the "analogue" of impulse is work: a product of force and distance. If force F acts on some object in the direction of motion and this object moves at distance l, work is defined as

$$A = Fl$$

First, let us check that dimension of work is the same as dimension of energy. Force is measured in Newtons and $1\ N = 1\ \dfrac{kg \cdot m}{s^2}$. Therefore the product of force and distance has dimension $\dfrac{kg \cdot m^2}{s^2}$ which is exactly the same as J which we defined as a unit of kinetic energy - unit of mass times square of unit of speed.

For momentum the equation for its change was simple: $\Delta p = J$ where Δp is the change of momentum and J is impulse. For energy there is a similar relation: $\Delta E = A$ but this formula will need clarification. Which forces should we account for when we calculate work? Is ΔE change of just the kinetic energy or total mechanical energy? We will answer these questions below.

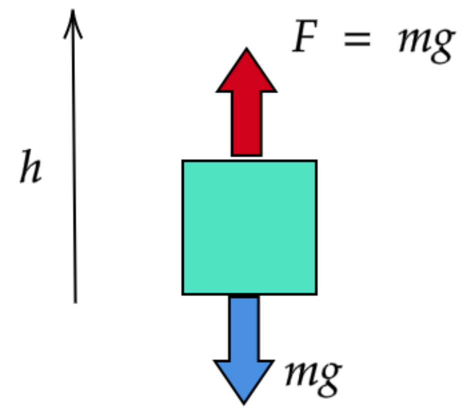
But before we refine the relation between change of energy and work, let us refine the formula for work as well. Recall that when we first defined work, we were only talking about forces in the same direction as displacement. What if they are not in the same direction? The answer is that then we should take displacement along the direction of force. If force is to the right, to calculate the work it only matters how far to the right an object moves. If it moves some distance l to the right then work of force F is $A = Fl$, as we discussed before. If it moves distance l to the left, it is the same as $-l$ to the right, so work becomes negative: $A = -Fl$. If it moves neither to the left nor to the right, just straight forward or backward, then work is just zero. So, if displacement makes 90° with force, work is 0. This is the reason why normal force does not change energy! Normal force is always at 90° to the surface and displacement us along the surface, so normal force does not perform any work.

Now let us return to the relation $\Delta E = A$. Let us look at an example: we lift a block of mass m to height h with the force F exactly equal to its gravity force mg so that it maintains its velocity. What work does force F perform? F is upwards, displacement h is also upwards, so

$$A_F = Fh = mgh$$

(as we take $F = mg$). What work did gravity perform? Now force mg is down but displacement is up, so

$$A_g = -mgh.$$



How did kinetic and potential energy change? Since net force on the block is zero, it moves with constant speed, so kinetic energy does not change:

$$\Delta E_{kin} = 0.$$

Potential energy increases by

$$\Delta E_{pot} = mgh.$$

Total mechanical energy therefore increases by

$$\Delta E_{mech} = \Delta E_{kin} + \Delta E_{pot} = mgh.$$

All these quantities are summarized at a table below.

A_F	A_g	ΔE_{kin}	ΔE_{pot}	ΔE_{mech}
mgh	$-mgh$	0	mgh	mgh

We see that if we want to write something like $\Delta E = A$ for kinetic energy only, we would have to take the work of all the forces. In this case total work

$$A_F + A_g = mgh - mgh = 0 = \Delta E_{kin},$$

and it is also true in general. So one way to write generally how energy changes in presence of forces is

$$\Delta E_{kin} = A_{all}, \tag{1}$$

where A_{all} is work of all the forces acting on our object. But it is not the only way to write it and the other way is often more convenient. Let us notice in our table that $\Delta E_{pot} = -A_g$. This turns out to be general and is actually how potential energy is properly defined (and not just postulated, as we did it). Let us take A_{all} and rewrite it as $A_{all} = A_g + A_{all-g}$ with A_{all-g} meaning work of all the forces other than gravity. Using this decomposition and $\Delta E_{pot} = -A_g$ our previous relation for change of kinetic energy could be transformed as

$$\Delta E_{kin} = A_{all} = A_g + A_{all-g} \Longrightarrow \Delta E_{kin} - A_g = \Delta E_{kin} + \Delta E_{pot} = \Delta E_{mech} = A_{all-g}.$$

The last relation is our desired result (called the work-energy theorem). We can see that our above example of lifting a block confirms it: $A_{all-g} = A_F = mgh$ in this case and $\Delta E_{mech} = mgh$ as well. Let us rewrite work-energy theorem once again clearly:

$$\Delta E_{mech} = A, \tag{2}$$

where A refers to work of all forces other than gravity. Let me stress that our derivation shows that equations (1) and (2) are completely equivalent but it is important not to confuse them. If we already accounted for the gravity force in energy by including potential energy we should not include the work of gravity force into the right hand side.

Why did we need to derive equation (2) if it's completely equivalent to equation (1)? It is often more convenient to use. From the definition of work it might seem that work performed by some force depends on the trajectory the object follows. For some forces, such as friction, it is certainly true. Such forces cannot be accounted for through the potential energy and their work always should be calculated explicitly. But for other forces, such as gravity, work only depends on the initial and final positions and it does not matter which trajectory the object takes between this points. This makes it possible to introduce potential energy which only depends on position. Calculating potential energy which depends just on the position is generally simpler than calculating work that a priori depends on the whole trajectory. This is why equation (2) is so convenient. Gravity could be neglected in this problem.

Homework

1. Find the work of the friction force which is necessary to stop a 1000 kg car moving at a speed of 72 km/h. Having found the work, use it to find braking distance. Friction coefficient is 0.1.

2. A water pump lifts 20 kg of water per second to the water supply tank which is 10 m over the ground level. What work is performed by the pump per 1 hour?

3. Compare the work done by a car's engine to accelerate the car from 0 km/h to 27 km/h with the work which is necessary to accelerate the car from 27 km/h to 54 km/h.

***4.** A beam of charged particles with different masses moves towards a region with constant electric field. It is not important to us what electric field is, the only thing we need to know is that in that region a constant force F acts on every particle (in the direction opposite to the initial motion). Width of this region is l. Speed of particles in the beam is the same and equal to v. What minimal mass m_0 should a particle have in order to get to the other side of the region with electric field? What will be the speed of the particle after it moves out of this region if its mass is m? Consider both the case $m > m_0$ and case $m < m_0$.