POTENTIAL ENERGY
JANUARY 17, 2021

Theory recap

Last time we discussed kinetic energy. We learned that any moving object with mass m and speed v has kinetic energy equal to $E_{kin} = \frac{mv^2}{2}$. But energy also exists in other forms, because we know situations when kinetic energy alone is not conserved. For instance, consider an object in free fall. Its speed is constantly increasing, so kinetic energy gets larger and larger. Where does the object get its kinetic energy from?

This question brings us to the concept of potential energy. The name suggests that an object with potential energy has a potential to gaining kinetic energy or transferring its energy to other objects. It is clear that the higher some object was initially, the greater will be its speed near the ground. Therefore potential energy should be bigger when the object is higher. The formula for potential energy is:

$$E_{potential} = mgh$$

Here m is mass of the object, g is free fall acceleration and h is the height above ground (we will return a bit later to the question if ground level is really important). So we see that heavier objects have larger potential energy and it also grows with height. It is very important that potential energy only depends on the current position of the object and does not depend on how the object got there.

We can introduce the concept of total mechanical energy, which is the sum of kinetic and potential energies:

$$E_{mechanical} = E_{kinetic} + E_{potential} = \frac{mv^2}{2} + mgh$$

Now we could state the law of conservation of mechanical energy. It applies in absence of certain forces, such as friction (we will have to wait until the next time for more details) and states that the total mechanical energy is conserved, so it does not change with time:

$$E_{mechanical} = const$$

For instance, we could check how energy conservation really works for an object in free fall. Let us start from a little exercise in kinematics. Assume that an object starts falling from rest at height h. We know that it will reach the ground in time t such that $\frac{gt^2}{2} = h$. On the other hand, its speed after this time t will be $v = gt$. Therefore we can express t via v: $t = \frac{v}{g}$ and insert it in our equation with h:

$$h = \frac{gt^2}{2} = \frac{g\left(\frac{v}{g}\right)^2}{2} = \frac{v^2}{2g} \Longrightarrow \frac{v^2}{2} = gh$$

Let us remember this formula for later and look at energy conservation. During the fall, potential energy is transferred into kinetic energy, but their sum (total mechanical energy) is conserved. So total mechanical energy at the beginning of the fall must be equal to the total mechanical energy near the ground. At the beginning kinetic energy was zero, as or object had no speed, but potential energy was mgh, so:

$$E_{mech,1} = E_{kin,1} + E_{pot,1} = 0 + mgh = mgh$$

On the other hand, near the ground kinetic energy is $\frac{mv^2}{2}$ where v is speed of the object there. Potential energy is now zero, because the object is at zero height. Therefore

$$E_{mech,2} = E_{kin,2} + E_{pot,2} = \frac{mv^2}{2} + 0 = \frac{mv^2}{2}$$

So, energy conservation law tells us that

$$E_{mech,1} = E_{mech,2} \Longrightarrow mgh = \frac{mv^2}{2}$$

Comparing it to our kinematical relation $\frac{v^2}{2} = gh$ we see that it is completely the same because mass cancels from both sides of the equation. So in this case we could calculate the final speed either way: from kinematics or from energy.

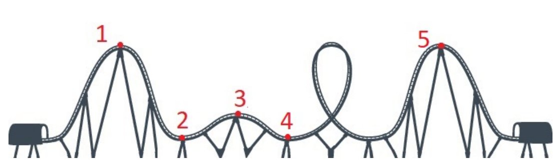
It is sometimes useful to rewrite the energy conservation law as a statement that change in kinetic energy plus change in potential energy is equal to zero:

$$E_{kin,1} + E_{pot,1} = E_{kin,2} + E_{pot,2} \Longrightarrow (E_{kin,2} - E_{kin,1}) + (E_{pot,2} - E_{pot,1}) = 0$$

In this form we can see that if we added the same number to both of the potential energies, their difference would stay the same. So the resulting change in kinetic energy would also stay the same. This actually means that it does not matter where do we measure the height from: if the reference point is shifted, all potential energies get shifted by the same amount and their difference (which is the only thing that really matters) stays the same.

Potential energy is not only useful when discussing free fall. Next time we will learn more what role forces other than gravity play for the energy conservation law. Until then, let us just accept as a fact that normal force does not spoil conservation of mechanical energy.

For instance, energy conservation could be applied to a roller coaster car if we neglect friction. Then the only forces acting on the car are gravity and normal force, so total mechanical energy is conserved. Let us analyze the motion of a car on the track on the figure to the right.



If a car has some speed v at point 1, as it goes down it will accelerate until it reaches point 2. After going through 2, the car will start to decelerate until point 3. Just by observing that 3 is lower than 1, we can immediately say that potential energy at point 3 is smaller than at point 1, so kinetic energy must be larger. Therefore speed at point 3 is larger than at point 1. After point 3, the car goes down again and accelerates. Point 4 is at the same height as point 2 - so potential energies at these two points are the same. Therefore kinetic energies are the same and speeds are equal. By the same logic speed at point 5 is equal the speed at point 1, as they are at the same height. We see that the concept of potential energy makes this analysis very simple and intuitive.

Finally, let us mention how one can store energy in the form of potential energy. A prominent example of this is a water power plant where huge amount of water is stored in an elevated reservoir. This water possesses potential energy which is released when we let it flow down. Its' potential energy is transferred first into kinetic energy of the flow, which is then converted into electric energy. But it is only possible because of the initial potential energy.

As a fun exercise, let us estimate how much potential energy is stored in the reservoir of the largest US water power plant, Grand Coulee in Washington. Roughly speaking, it contains 10 cubic kilometers of water, or 10 billion tons of water. This water is elevated at a height of approximately 100 m. Therefore, potential energy stored could be roughly estimated as

$$mgh = 10^{10}\ tons \cdot 10\ \frac{m}{s^2} \cdot 100\ m = 10^{13} \cdot 10 \cdot 100\ J = 10^{16} J$$

To get an idea, how much energy this is, let us mention that an average household consumes about 40 gigajoules, or $4 \cdot 10^{10}\ J$, per year. Therefore, if potential energy stored in the reservoir of Grand Coulee was fully converted into electricity, it would be enough to supply $10^{16}/(4 \cdot 10^{10}) = 250,000$ houses with electricity for a whole year. This is about a quarter of the amount of houses on Long Island!

Homework

1. A 10 g bullet is sent up at a speed of 300 $\frac{m}{s}$. How high it will go? Solve this problem in two ways (through kinematics and through energy conservation).

2. A 50 g ball is falling down. As the ball passes a certain distance its potential energy changes by 2 J. Calculate this distance. Does this distance depend on the initial velocity of the ball?

3. A cyclist initially at rest goes downhill from the top of 20 m hill. Assuming that only 50% of potential energy is converted into kinetic energy (with the rest being lost to friction and air resistance), find speed of the cyclist be at the bottom of the hill.

***4.** Find potential energy of a thin rod of mass m and length l which is placed vertically on the floor (height is measured from the floor).