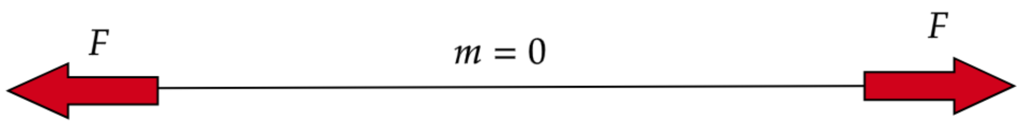
**ROPES AND PULLEYS**

DECEMBER 6, 2020

## Theory recap

Now that we have discussed several different kinds of forces, it is time to have some fun with a bit more complicated mechanical systems. They may include several bodies connected by ropes, springs and maybe using pulleys. Today we discuss the properties of these various elements and how they can be used to give one an advantage in force.

Let us begin with ropes. We will use an approximation that ropes have no mass. Practically it is a good approximation if all the blocks in the system are much heavier than the ropes. It leads to an important consequence: a massless rope has the same tension in every point. This is because for a given piece of the rope net force acting on it from the two adjacent pieces must be zero by Newton's second law, because $ma = 0$ for $m = 0$.



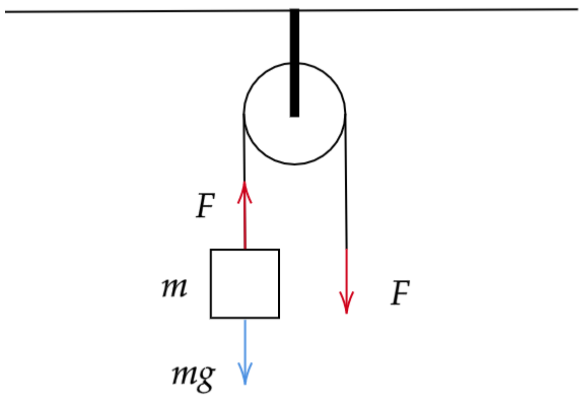

Actually, the same is usually assumed for springs: we assume that they are massless so the elastic force is the same everywhere in the spring. There is one important distinction between ropes and springs though: ropes can only pull because they are soft, but springs are rigid and they can both push and pull.

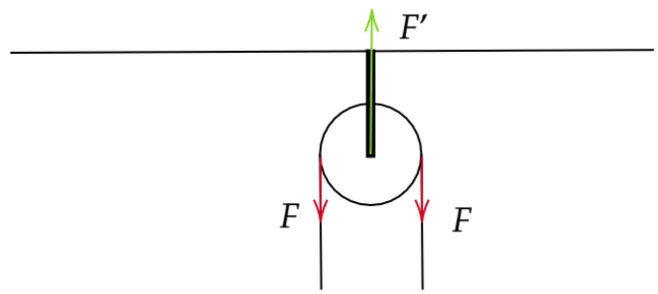
Suppose you want to move some block up but it is convenient for you to pull the rope down. Could you somehow arrange it? The answer is yes! You just need to wrap the rope around something so that its' direction is changed. We want to wrap the rope around something circular and fixed to stay in one place. Such an object is called a fixed pulley.

Let us consider a fixed pulley attached to the ceiling (see the figure below). On one side of the pulley there is a block of mass $m$ which is at rest and on the other side we pull the rope down with force $F$. What is $F$ equal to? Remember our discussion that a straight massless rope "transmits" tension fully. If the rope goes around a pulley it remains true if we neglect friction between the rope and the pulley. It means that the rope pulls the block with the same force $F$ as we pull the rope. Since the block is at rest, the net force acting on it must be zero. Therefore

$$F - mg = 0 \implies F = mg$$

So we have to pull with the same force but the convenience is that we could pull the rope down and still be able to hold the block. So fixed pulley is used to redirect the force but will not give us any force advantage. But there is a way to rearrange the system in such a way that we will get a force advantage.



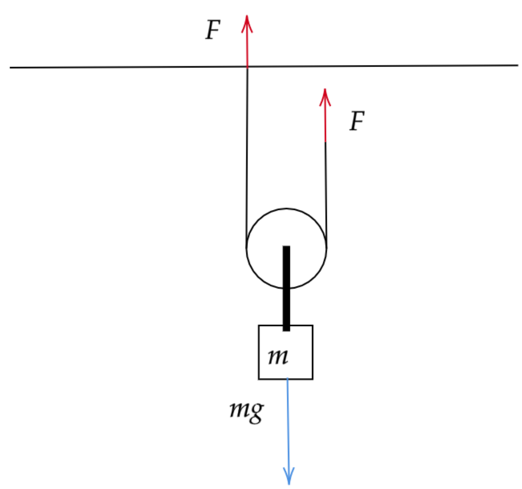




To get a hint on how one could get a force advantage, let us first take a look at the force which the fixed pulley exerts on the ceiling. From the figure above to the right we could see that the pulley itself interacts with two pieces of rope (one on each side of the pulley) and with the ceiling. As we have learned, tension of the rope is the same everywhere, so each of the forces with which ropes act on the pulley is $F$ and is directed down. Calling the force which acts on the pulley upwards from connection to the ceiling $F'$ we may write the net force acting on the pulley as

$$F_{net} = F' - 2F = 0 \Longrightarrow F' = 2F$$

We have used the fact that the pulley is in equilibrium so net force acting on it is zero.

The trick on how to get a force advantage is to turn the pulley upside down and let it move (like on the figure to the right). Now our block is attached directly to the pulley and they move together. The rope is at one end attached to the ceiling and we pull the other end with some force $F$. We will assume that the pulley has mass zero, so the net force on it must always be zero (the same way as for the massless rope). This kind of pulley is called moving pulley. By looking at moving pulley as an upside down fixed pulley (with $mg$ now playing the role of $F'$) one could deduce that




$$mg = 2F \implies F = \frac{mg}{2}$$

If you feel suspicious about the analogy, an independent derivation is also possible. The simplest way is to look at the system of pulley and block together. It is in equilibrium, so it has zero acceleration and the net force must be zero. There are three external forces acting on this system: two pieces of rope each pull up with the same force $F$ and gravity force $mg$

acts down. Adding all three forces with the correct signs accounting for directions we get the net force:

$$F_{net} = F + F - mg = 2F - mg = 0 \implies F = \frac{mg}{2}$$

So a moving pulley gives us a force advantage: we just have to pull with half the gravity force in order to support a block. For example if you could hold up to 30 $kg$ with bare hands, you could hold up to 60 $kg$ with a moving pulley! At first this might sound a bit mysterious: where does the other half of the force come from? As we could see from the above figure, the other half is supplied by the ceiling to which the rope is attached.

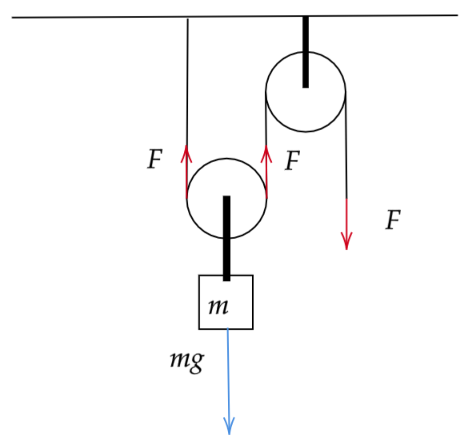
## Additional material not discussed in class

This is a small section discussing pulleys a bit further than we had time for in the class.

The true power and beauty comes when we start combining pulleys to achieve our practical goals (convenience of operation and particular force advantage). In the homework you will have a problem on inventing a system that gives force advantage factors 3 and 4, but in fact any desired natural number factor could be achieved, at least theoretically.

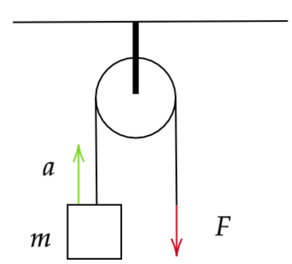
The simplest combination of two pulleys is a system built out of a moving and a fixed pulley with one rope going around both of them. It combines the force advantage of 2 (provided by the moving pulley) with the convenience of pulling down (provided by the fixed pulley). Let us prove that it has the same force advantage as a single moving pulley. If we pull the rightmost piece of the rope with force $F$, tension force everywhere in the rope is $F$. So two pieces of the rope pull the moving pulley upwards with force $F$ each, as before. Therefore

$$2F = mg \implies F = \frac{mg}{2}$$

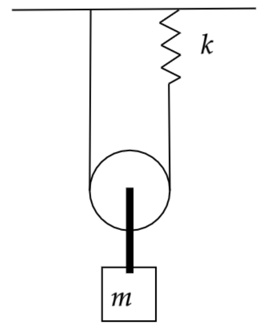



## Homework

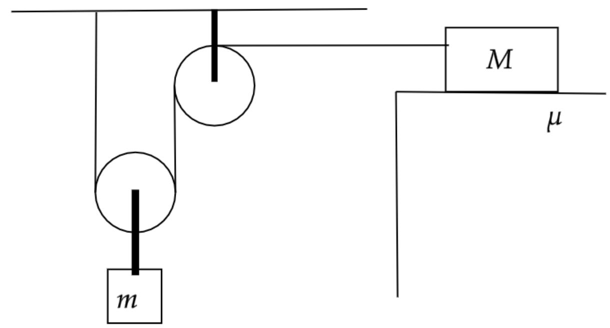
**1.** You have built a mechanical system out of a frictionless fixed pulley and a massless rope. With what force should you pull the rope down in order to move a block of mass $m = 5\ kg$ with acceleration $a = 2\ m/s^2$ up?



**2.** A massless moving pulley is used to hold a block of mass $m = 20\ kg$. One of the ends of the rope is attached directly to the ceiling and the other is attached to the ceiling via a spring with spring constant $k = 5\ N/cm$. Find elongation of the spring in equilibrium. How does it compare to elongation we would get if this block was suspended from this spring directly?




***3.** A fixed pulley and a massless moving pulley are assembled into a system sharing one massless rope. A block of mass $m = 3\ kg$ is attached to the moving pulley. The free end of the rope is attached to a block of mass $M = 20\ kg$ placed on a horizontal surface with some friction. Find the friction coefficient $\mu$ if we know that block $M$ moves with constant velocity because of the rope pulling it.




***4.** Draw a system that gives a force advantage 4 : so you have to pull the rope with a force $\dfrac{mg}{4}$ in order to support a block of mass $m$. You could use as many fixed and moving pulleys and ropes as you'd like. Prove that the system you drew gives a force advantage 4. *Hint: Try using two moving pulleys*

***5.** Same problem as above but for force advantage 3.