Homework 11.

Momentum

Momentum is the product of the mass and velocity of an object. Momentum is a vector: it has both a direction and a magnitude.

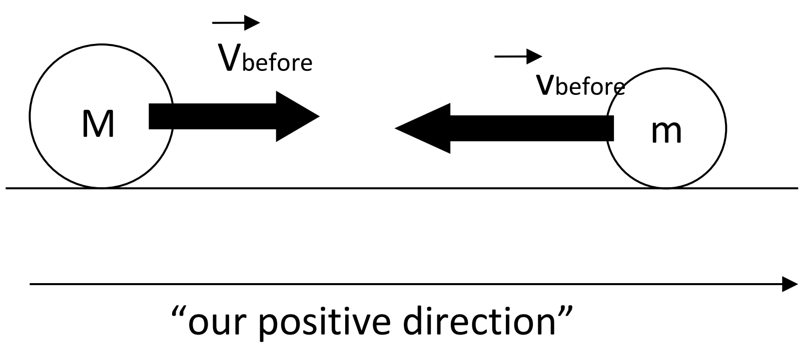
$$\vec{P} = m\vec{V}$$

Momentum is measured in *kg m/s*. As we can see, we can change the momentum by changing the velocity or the mass of an object. To change the velocity, we need to apply force. To change the mass of a moving object we could, for example, put an additional load to the rolling cart.

As we can see, if no forces applied to an object (or objects) and total mass of the object (objects) does not change, the momentum does not change as well.

If an object or a group of objects are isolated (which means that there is no external force applied to them) the total momentum of this object or group of objects does not change no matter how strong they interact with each other. In this case we can say that the total net momentum of the objects *conserves*.

Example: Find total net momentum of two balls with masses *m* and *M* and velocities V and v rolling toward each other.



Solution: First we will choose "positive direction". The momentum and velocity of an object have same direction. I picked up "right-to-left", but the result (as we know) will not change if we choose "left-to-right". The momentum of the left ball is: $P=MV_{before}$, "before" here means "before the collision". We consider it as positive, because the momentum (as the velocity) "looks" in our "positive" direction. The momentum of the left ball p= - mv_{before}. It is negative, because it "looks" in the opposite direction. So, the total net momentum, $\boldsymbol{P_{before}}$ is:

$$P_{before} = MV_{before} - mv_{before}$$

If $\boldsymbol{MV_{before}}$ is larger than $\boldsymbol{mv_{before}}$, the total net momentum is positive. It means that it is directed "positively" - left-to-right. If $\boldsymbol{MV_{before}}$ is less than $\boldsymbol{mv_{before}}$, the total net momentum is negative and "looks" right-to-left.

Now let us assume that after the collision the right ball bounces back and moves in opposite directions. In this case total momentum after the collision is:

$$P_{after} = MV_{after} + mv_{after}$$

We do not know where and how fast will be moving the right ball. But we can find it out using the momentum conservation law. As long as no external force is applied to the balls the total momentum of the balls does not change (conserves) and its value before the collision will be equal to this after the collision:

$$P_{before} = P_{after}$$

$$MV_{before} - mv_{before} = MV_{after} + mv_{after}$$ from this we obtain:

$$MV_{before} - mv_{before} - mv_{after} = MV_{after},$$ and finally:

$$V_{after} = V_{before} - \frac{m}{M}(v_{before} + v_{after})$$

The sign V_{after} will tell us where does the right ball move after the collision: if it is "+" then the ball will continue moving to the right, if it is "-" then the ball will bounce back.

Problems:

1. Two cars with mass 1000kg and 2000kg go toward each other. The speed of the first car is 50km/h, the speed of the second is 40 km/h. Find the total net momentum of the two cars. Make a picture.

2. A 80kg jogger runs with a constant acceleration of 1/5 m/s^2 for 10 seconds. How his momentum changed during this time?

4. A 10kg ball moving at a speed of 10m/s hits a 5kg ball which was at rest before the collision. After the collision the smaller ball starts moving at a speed 10m/s. Find the velocity of the heavy ball after the collision.

5. A fox is chasing a small rabbit. The momentum of the fox is equal to the momentum of the rabbit. Will the fox catch the rabbit?

6. You send a 100g ball up and it returns back in 6 seconds. Find the initial momentum of the ball and its momentum in the highest point.