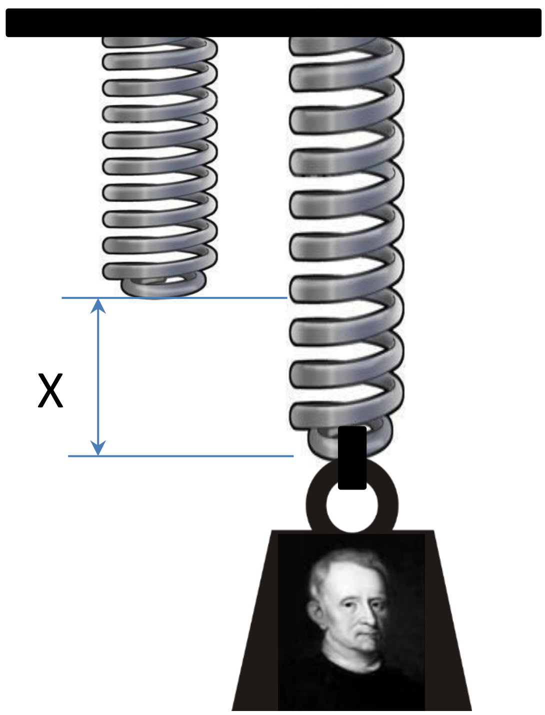
Homework 9

Elastic force

Last class we discussed elasticity and elastic force. Elasticity can be described as the property of an object to return to its original shape after the force, which caused deformation of the object, is lifted. A lot of objects around us show elastic properties. When you are knocking on the door, for example, the door and your knuckles experience small elastic deformation. We will discuss elastic force on the example of a coil spring. As you are stretching or squeezing a coil spring, the spring applies the “resistance” force to your hand. The force “tries” to return the spring to its original length. This force is called *elastic* force:

$$\vec{\mathbf{F}} = -k\vec{\mathbf{x}} \quad (1),$$

where ***x*** is the extension of the coil spring which means the displacement of the end of the coil spring from its initial position (see picture below). The latter is the position where the spring would naturally come to rest. The coefficient ***k*** in the formula is called *force constant* of *spring constant*. The constant has units of force per unit length (usually newtons per meter). This constant depends only of the coil spring material and shape.



Robert Hooke (1635-1703)

Why minus in the formula? This is because the elastic force is directed against displacement. If we squeeze the coil spring, the elastic force “tries” to stretch it back. The formula (1) is also called the Hooke’s law. Hooke's law is named after the 17th century British physicist Robert Hooke.

He first stated this law in 1676 as an anagram, then in 1678 in Latin as *Ut tensio, sic vis*, which means: “As the extension, so the force”. For objects that obey Hooke's law (we will call them elastic, such as coil springs or, say, rubber bands), the force which “tries” to return the coil spring back to the initial length is proportional to the extension x.

All the objects change their shape or, using other words, their “lengths” along different directions under applied force. We will call this change as “deformation”. In the case of coil spring the deformation is the change of the coil spring's length Δx. Even in case of a very low force (say, touch of the finger) applied to a really hard object (say, a diamond or a steel ball) the latter slightly deforms. We cannot see this deformation, but it is finite and measurable.

The deformation is called ***elastic*** if the deformed object restores its original shape after the force is removed. We can demonstrate elastic deformation by squeezing a rubber ball or stretching a coil spring. If the deformation stays after the force is lifted it is called ***plastic***. The examples are spreading butter over bread or making something from play dough.

Each object has its deformation limit. If you exceed this limit when applying force to the object the latter breaks. Even if the object is pretty hard (which means its force constant k is high) it may have low deformation limit. We call such objects ***brittle*** and the corresponding quality as ***brittleness***. The examples of brittle materials are glass and porcelain. The opposite qualities are ***ductility*** and ***malleability***. Ductile materials can be strongly deformed when stretched, malleable can be strongly deformed when compressed. An example of both ductile and malleable material is chewing gum.

Problems

1. Take a piece of any elastic thread. Make marks on the thread separated by equal distances along the entire length of the thread (you can use a pen or marker). Then, stretch the thread. The distances between the marks will increase. Will the distances remain equal to each other? Try to explain the result.

2. Measure the force constant of the rubber band used for the problem 1.

3. A raw egg is very easy to break but extremely difficult to crush with one hand (if you try to do this experiment, please, be very careful!). Is there a contradiction? How you would characterize the material of an eggshell?

4. A load of 1kg is attached to the end of the coil spring hanging from the ceiling of an elevator. The force constant of the spring is 100N/m. The elevator goes up with the acceleration of the 10m/s2. Find the extension of the coil spring.

5. Solve the problem 4 for the case where the elevator goes down with the acceleration of $30 m/s^2$. What will happen to the coil spring?