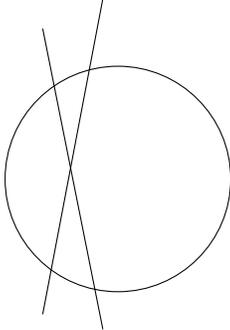


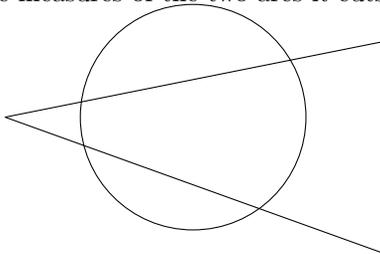
MATH 9: HOMEWORK 8
NOVEMBER 15, 2020 (DUE NOV 22)

1. GEOMETRY PROBLEMS

1. Given an angle inside a circle, consider the angle and its vertical angle, and the two arcs they cut off. Prove that the measure of the original angle is equal to half the sum of the angle measures of the two arcs cut off by the angle and its vertical.



2. Similar question but for an angle situated outside a circle. Express, with proof, the angle in terms of the measures of the two arcs it cuts off.



3. A point P is chosen on the chord AB of a circle with the center O and radius R . Show that $|AP| \times |PB| = R^2 - |OP|^2$.
4. Consider all triangles with a given base and given altitude corresponding to this base. Prove that among all these triangles the isosceles triangle has the biggest angle opposite to the base.
5. In a *trapezoid* $ABCD$ with bases AB and CD , a line passing through vertex D passes through a point E on the side AB , such that $|AE|$ is $1/n$ -th of $|AB|$, where n is a positive integer. Additionally, the ratio of the bases $|AB| : |CD|$ is m , where m is a positive integer. At what distance from A , relative to the length, $|AC|$, of the diagonal AC it meets this diagonal?

2. ALGEBRA PROBLEMS

1. Using the method of mathematical induction, prove the following equality,

$$\sum_{k=0}^n k \cdot k! = (n+1)! - 1$$

2. Put the sign $<$, $>$, or $=$, in place of \star in: $\frac{n+1}{2} \star \sqrt[n]{n}$

3. Find the value of the continued fraction given by $\{1, 2, 3, 3, 3, \dots\} = 1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \dots}}}}$

4. Explain why \mathbb{N} is countable. Then explain whether \mathbb{Z} is countable.
5. It is known that the set of all real numbers is not countable. Let us consider the following set $S = \{x : 0 \leq x \leq 1\}$. Is this set countable? Can you prove it?