

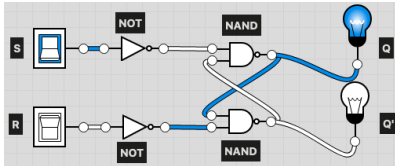
MATH 8: HANDOUT 8
LOGIC 3: SR FLIP-FLOPS. CONDITIONALS

SR FLIP-FLOP

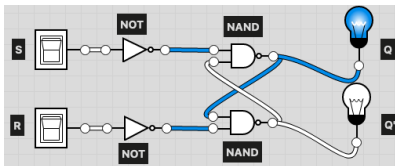
Today we played a little more with circuits, and especially with SR-Flip Flop. Consider the following circuit, which is called SP-Flip Flop. Interestingly, the output of NAND-gates is also an input to other NAND-gates. Let us look at how it works.

Let's imagine that we turn S on to 1. NOT-gate changes it to 0, and when it is fed to the NAND gate, the output of it would be 1, since $0 \text{ NAND } X = 1$ for any X (make sure you understand why it is so!). As a result, Q bulb will turn on.

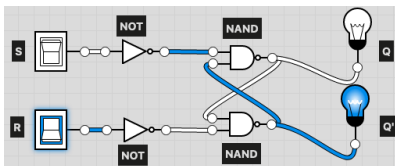
At the same time, R is off (is 0), and it is changed to 1 by the NOT-gate, and fed to NAND along with the output of the top NAND-gate, so the output of the bottom NAND-gate is 0 (since $1 \text{ NAND } 1 = 0$).



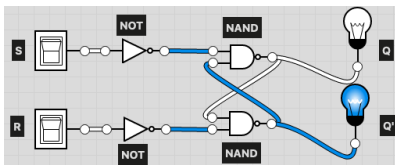
Interestingly, if we now flip S off, the lightbulbs will not change their state: lightbulb Q will stay on, and Q' will stay off: one of the inputs to the top NAND will always stay off, regardless of what S is.



Now if we switch S to off, and turn R on, the lightbulbs will flip: Q' will be lit up, and Q will be off.



We will also observe a similar situation: now switching R to off will not change the state of lightbulbs:



The state when the top lightbulb is lit up is called S-state, that is the flip-flop is in SET position. When the lower lightbulb is on, the flip-flop is in RESET (R) position.

The interesting thing about this circuit is that it has **memory**: once it's in SET-state, the top lightbulb indicates it, and switching S-switch off won't change anything – the top lightbulb will still be on. Similarly, once we're in R-state, we can switch R-switch off, but the lightbulbs will still indicate that we are in the R-state (the 2nd light bulb is on)

CONDITIONAL

In addition to all previous logic operations, there is one more which we have not yet fully discussed: **implication**, also known as **conditional** and denoted by $A \implies B$ (reads *A implies B*, or “If *A*, then *B*”). It is defined by the following truth table:

A	B	$A \implies B$
T	T	T
T	F	F
F	T	T
F	F	T

Note that in particular, in all situations where A is false, $A \implies B$ is automatically true. E.g., a statement “if $2 \times 2 = 5$, then...” is automatically true, no matter what conclusion one puts in place of dots.

Another logic operation is called **equivalence** and defined as $(A \iff B)$ is true if A, B always have the same value (both true or both false).

One can easily see that $(A \iff B)$ is equivalent to $(A \implies B) \text{ AND } (B \implies A)$.

PROBLEMS

1. Show that $A \implies B$ is not equivalent to $B \implies A$, i.e. they have different truth tables and one of them can be true while the other is false.
2. Prove the contrapositive law: $A \implies B$ is equivalent to $(\neg B) \implies (\neg A)$
3. Show that $(A \implies B)$ is equivalent to $B \vee \neg A$. Can you rewrite $\neg(A \implies B)$ without using implication operation?
4. Consider the following statement (from a parent to his son):
 “If you do not clean your room, you can’t go to the movies”
 Is it the same as:
 - (a) Clean your room, or you can’t go to the movies
 - (b) You must clean your room to go to the movies
 - (c) If you clean your room, you can go to the movies
5. English language (and in particular, mathematical English) has a number of ways to say the same thing. Can you rewrite each of the verbal statements below using basic logic operation (including implications), and variables
 - A : you get score of 90 or above on the final exam
 - B : you get an A grade for the class
 (As you will realize, many of these statements are in fact equivalent)
 - (a) To get A for the class, it is required that you get 90 or higher on the midterm
 - (b) To get A for the class, it is sufficient that you get 90 or higher on the midterm
 - (c) You can’t get A for the class unless you got 90 or above on the final exam
 - (d) To get A for the class, it is necessary and sufficient that you get 90 or higher on the midterm
6. Show that in all situations where A is true and $A \implies B$ is true, B must also be true. [This simple rule has a name: it is called *Modus Ponens*.]
7. Show that if $A \implies B$ is true, and B is false, then A must be false. [This is called *Modus Tollens*.]