MATH 7: HANDOUT 3
ALGEBRAIC EXPRESSIONS AND IDENTITIES

Main Algebraic Identities

Here is a list of the main algebraic identities we discussed:

1. $(ab)^n = a^n b^n$
2. $\sqrt{ab} = \sqrt{a}\sqrt{b}$
3. $(a+b)^2 = a^2 + 2ab + b^2$
4. $(a-b)^2 = a^2 - 2ab + b^2$
5. $a^2 - b^2 = (a-b)(a+b)$

Replacing in the last equality a by $\sqrt{a}$, b by $\sqrt{b}$, we get

$$(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a - b$$

which is very helpful in simplifying expressions with roots, for example:

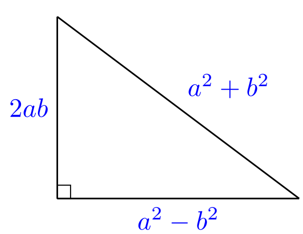
$$\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2} - 1$$

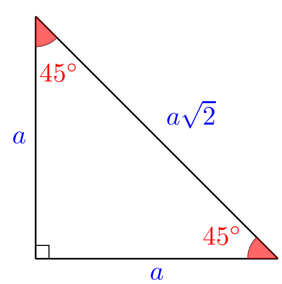
We also discussed solving simple equations: linear equation (i.e., equation of the form $ax+b=0$, with a, b some numbers, and x the unknown) and equation where the left hand side is factored as product of linear factors, such as $(x-2)(x+3) = 0$.

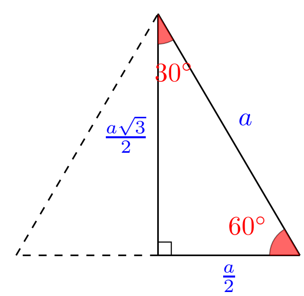
Pythagoras' Theorem

In a right triangle with legs a and b, and hypotenuse c, the square of the hypotenuse is the sum of squares of each leg. $c^2 = a^2 + b^2$. The converse is also true, if the three sides of a triangle satisfy $a^2 + b^2 = c^2$, then the triangle is a right triangle. Some Pythagorean triples are: (3,4,5), (5,12,13), (7,24,25), (8.15,17), (9,40,41), (11,60,61), (20,21,29).

To generate such Pythagorean triples, choose two positive integers a and b. Then plug the values into the sides as shown on the first picture:







Try to figure out why the sides of this triangle satisfy the Pythagoras' Theorem!

45-45-90 Triangle: If one of the anglesin a right triangle is $45°$, the other angle is also $45°$, and two of its legs are equal. If the length of a leg is a, the hypothenuse is $a\sqrt{2}$.

30-60-90 Triangle: If one of the angles in a right triangle is $30°$, the other angle is $60°$. Such triangle is a half of the equilateral triangle. That means that if the hypothenuse is equal to a, its smaller leg is equal to the half of the hypothenuse, i.e. $\frac{a}{2}$. Then we can find the other leg from the Pythagoras' Theorem, and it will be equal to $\frac{a\sqrt{3}}{2}$.

Homework

1. Simplify

(a) $\frac{42^2}{6^2} =$

(b) $\frac{6^3 \times 6^4}{2^3 \times 3^4} =$

(c) $(2^{-3} \times 2^7)^2 =$

(d) $\frac{3^2 \times 6^{-3}}{10^{-3} \times 5^2} =$

2. Simplify

(a) $\frac{a}{2} + \frac{b}{4} =$

(b) $\frac{1}{a} + \frac{1}{b} =$

(c) $\frac{3}{x} + \frac{5}{xy} + \frac{5}{3a} =$

3. Using algebraic identities calculate

(a) $299^2 + 598 + 1 =$

(b) $199^2 =$

(c) $51^2 - 102 + 1 =$

4. Expand

(a) $(4a - b)^2 =$

(b) $(a + 9)(a - 9) =$

(c) $(3a - 2b)^2 =$

5. Factor

(a) $ab + ac =$

(b) $3a(a + 1) + 2(a + 1) =$

(c) $36a^2 - 49 =$

6. Write each of the following expressions in the form $a + b\sqrt{3}$, with rational a, b:

(a) $(1 + \sqrt{3})^2$

(b) $(1 + \sqrt{3})^3$

(c) $\frac{1}{1 - 2\sqrt{3}}$

(d) $\frac{1 + \sqrt{3}}{1 - \sqrt{3}}$

(e) $\frac{1 + 2\sqrt{3}}{\sqrt{3}}$

7. In a trapezoid ABCD with bases AD and BC, $\angle A = 90°$, and $\angle D = 45°$. It is also known that $AB = 10$ cm, and $AD = 3BC$. Find the area of the trapezoid.

8. In a right triangle ABC, BC is the hypotenuse. Draw AD perpendicular to BC, where D is on BC. The length of BC=13, and AB=5. What is the length of AD?

9. Factor

(a) $ab + ac =$

(b) $3a(a + 1) + 2(a + 1) =$

(c) $36a^2 - 49 =$

10. What is the area of a regular hexagon whose side is 5cm?