

Homework 11: More on Binomial probabilities

HW11 is Due December 20; submit to Google classroom 15 minutes before the class time.

1. Binomial probabilities

The binomial coefficients are also useful in calculating probabilities. Imagine that we have some event that happens with probability p ("success") and does not happen with probability $q = 1 - p$ ("failure"). Then the probability of getting k successes in n trials is:

$$P(k \text{ successes in } n \text{ trials}) = \binom{n}{k} p^k q^{n-k}$$

Where,

n — number of trials; **trial**: one instance of an experiment. For example, if we are doing a sequence of coin tosses, each coin toss is a trial. If we are shooting ducks, each shot is a trial

- k — number of successes; **success**: a trial that ends up in a desired outcome. If we are looking for Heads, *success* is an outcome of getting a Head. If we are looking at duck shooting, *success* is a hit.
- $n - k$ — number of failures. **failure**: a trial that does not end up in a success (missing a duck, getting a Tail while looking for Heads)
- p — probability of success in one try; **example**: probability of success in one trial (1/2 for heads/tails, 1/6 to roll any number on a die)
- $q = 1 - p$ — probability of failure in one try; **example**: probability of failure in one trial (1-1/2= 1/2 for heads/tails, 1-1/6 = 5/6)

Example: You roll a die 100 times. What is the probability of getting a 6 exactly 20 times?

Solution: Here we roll the die $n = 100$ times, we got a 6 $k = 20$ times, where the probability for rolling a 6 is $p = 1/6$, and the probability for not rolling a 6 is $q = 5/6$. Then using the binomial probability formula, we calculate the probability as:

$$P = \binom{100}{20} \left(\frac{1}{6}\right)^{20} \left(\frac{5}{6}\right)^{80} = \binom{100}{20} \times \frac{5^{80}}{6^{100}}$$

Example: A hunter is shooting ducks. Probability of hitting a duck with one shot is $p = 1/3$. What is the probability that out of 7 shots, she will hit exactly three times?

Solution: Here we have $n = 7$, $k = 3$, $p = 1/3$, $q = 2/3$. Then

$$P = \binom{7}{3} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{7-3} = \binom{7}{3} \times \frac{2^4}{3^7}$$

Homework problems

Instructions: Please always write solutions on a **separate sheet of paper**. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

Note: In the problems below, you can give your answer as a binomial coefficient without calculating it. If you want to calculate it, use the Pascal's triangle to find the value of $\binom{n}{k}$, where k is the k -th element in the n -th row of the Pascal triangle, counting from 0.

1. A (blindfolded) marksman finds that on the average he hits the target 4 times out of 5. If he fires 4 shots, what is the probability of
 - (a) more than 2 hits?
 - (b) at least 3 misses

2. In each of 4 races, the Democrats have a 60% chance of winning. Assuming that the races are independent of each other, what is the probability that:
 - (a) The Democrats will win 0 races, 1 race, 2 races, 3 races, or all 4 races?
 - (b) The Democrats will win at least 1 race.
 - (c) The Democrats will win a majority of the races.

3. The ratio of boys to girls at birth in Singapore is quite high at 1.09 : 1.
What proportion of Singapore families with exactly 6 children will have at least 3 boys? 9HI

4. **(Same as problem 4 from HW 10)** A hunter is shooting ducks. Probability of hitting a duck with one shot is $p = 1/3$.
 - (a) The hunter makes 5 shots. What is the probability that she misses all five?
 - (b) What is the probability that out of 5 shots, she will hit a duck at least once? Will this probability double if she makes 10 shots? (You can use the calculator for computing the answers)
 - (c) What is the probability that out of 5 shots, she will hit exactly once? Will this double if she makes 10 shots?
 - (d) What is the probability that out of 5 shots, she will hit a duck exactly three times? Will this probability double if she makes 10 shots? (You can use the calculator for computing the answers)
 - (e) What is the probability that she hits a duck half the time or more if she fires 5 times (i.e. 3, 4, or 5 hits)? What about if she fires 10 times (i.e. 5, 6, 7, 8, 9, or 10 hits)?
 - (f) What is the most likely number of hits out of 5 shots? And out of 10 shots?

5. **(Same as problem 5 from HW 11)** At a fair, they offer you to play the following game: you are tossing small balls in a large crate full of empty bottles; if at least one of the balls lands inside a bottle, you win a stuffed toy (worth about \$5). Unfortunately, it is really impossible to aim, so the game is just a matter of luck (or probability theory): every ball you toss has a 20% probability of landing inside the bottle.
 - (a) If you are given three balls, what is the probability that all three will be hits? That all three will be misses? That at least one will be a hit?
 - (b) Same questions for five balls.
 - (c) What about seven balls?
 - (d) How much should the organizers charge for 3 balls to break even? What about for 5 balls?