Homework 4: Algebraic identities – summary. Pythagoras theorem.

HW5 is Due November 1st; submit to Google classroom 15 minutes before the class time.

Here are some of the basic algebraic identities we have discussed today and used to solve problems:

1. **Exponents Laws**

   If a *and* b are real numbers and n is a positive integer

   $(ab)^n = a^n b^n$ (eq. 1)

   $\sqrt{ab} = \sqrt{a}\sqrt{b}$ (eq. 2)

   $(a+b)^2 = a^2 + 2ab + b^2$ (eq.3)

   $(a-b)^2 = a^2 - 2ab + b^2$ (eq. 4)

   And also: $a^2 - b^2 = (a-b)(a+b)$ (eq. 5)

   Replacing in the last equality ***a*** by $\sqrt{\boldsymbol{a}}$, **b** by $\sqrt{\boldsymbol{b}}$, we get : $a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$ (eq. 6)

2. **Simplifying expressions with roots** (rational expressions)

   The above identity (eq. 6) can be used to simplify expressions with roots by expanding the fractions with a term which "removes" the roots from the denominator:

$$\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{\left(\sqrt{2}\right)^2 - 1^2} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

3. **Quadratic equations of a specific form**

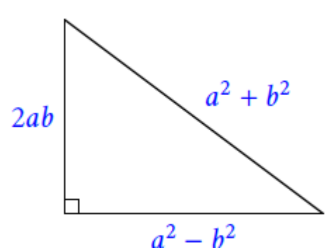
   We also discussed solving simple equations:

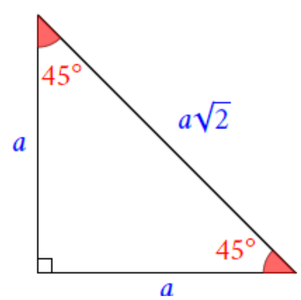
   - linear equation (i.e., equation of the form $ax + b = 0$, with a, b some numbers, and x the unknown and equation)
   - two types of quadratic equations (i.e, equations where the unknown is squared, x^2) when the left-hand side could be factored as product of linear factors, i.e, $(x-2)(x+3) = 0$.

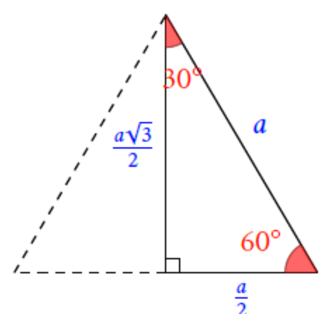
4. **Pythagoras' theorem**

   In a right triangle with legs ***a*** and ***b***, and hypotenuse ***c***, the square of the hypotenuse is the sum of squares of each leg: $c^2 = a^2 + b^2$. The converse is also true, if the three sides of a triangle satisfy $c^2 = a^2 + b^2$, then the triangle is a right triangle. Some Pythagorean triples are: (3,4,5), (5,12,13), (7,24,25), (8.15,17), (9,40,41), (11,60,61), (20,21,29).

   To generate such Pythagorean triples, choose two positive integers a and b. Then plug the values into the sides as shown on the first picture:







Try to figure out again why the sides of this triangle satisfy the Pythagoras' Theorem!

45-45-90 Triangle: If one of the angles in a right triangle is 45^0, the other angle is also 45^0, and two of its legs are equal. If the length of a leg is *a*, the hypothenuse is $a\sqrt{2}$.

30-60-90 Triangle: If one of the angles in a right triangle is 30^0, the other angle is 60^0. Such triangle is a half of the equilateral triangle. That means that if the hypothenuse is equal to *a*, its smaller leg is equal to the half of the hypothenuse, i.e. $\frac{a}{2}$. Then we can find the other leg from the Pythagoras' Theorem, and it will be equal to $\frac{a\sqrt{3}}{2}$.

Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Simplify
   a. $\frac{42^2}{6^2} =$
   b. $\frac{6^3 \times 6^4}{2^3 \times 3^4} =$
   c. $(2^{-3} \times 2^7)^2 =$
   d. $\frac{3^2 \times 6^{-3}}{10^{-3} \times 5^2}$

2. Simplify
   a) $\frac{a}{2} + \frac{b}{4} =$
   b) $\frac{1}{a} + \frac{1}{b} =$
   c) $\frac{3}{x} + \frac{5}{xy} + \frac{5}{3a} =$

3. Using algebraic identities calculate
   a. $299^2 + 598 + 1 =$
   b. $199^2 =$
   c. $51^2 - 102 + 1 =$

4. Expand
   a. $(4a - b)^2 =$
   b. $(a + 9)(a - 9) =$
   c. $(3a - 2b)^2 =$

5. Factor (i.e., write as a product) the following expressions:
   a. $ab + ac =$
   b. $3a(a + 1) + 2(a + 1) =$
   c. $36a^2 - 49 =$

6. Write each of the following expressions in the form $a + b\sqrt{3}$ with rational a, b. (No root in the denominator):
   a. $(1 + \sqrt{3})^2$
   b. $(1 + \sqrt{3})^3$
   c. $\frac{1}{1 - 2\sqrt{3}}$
   d. $\frac{1+\sqrt{3}}{1-\sqrt{3}}$
   e. $\frac{1+2\sqrt{3}}{\sqrt{3}}$

7. In a trapezoid ABCD with bases AD and BC, $\angle A = 90^0$, and $\angle D = 45^0$. It is also known that AB = 10 cm, and $AD = 3BC$. Find the area of the trapezoid.

8. In a right triangle ABC, BC is the hypotenuse. Draw AD perpendicular to BC, where D is on BC. The length of BC=13, and AB=5.What is the length of AD?